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RESEARCH ARTICLE

Rolling Bearing Remaining Useful Life Prediction Based on Wiener Process

Wentao Zhao,[1,2] Chao Zhang,[1,2] Shuai Wang,[1,2] Da Lv,[1,2] and Oscar García Peyrano[3]
[1]School of Mechanical Engineering, Inner Mongolia University of Science and Technology, Baotou 014010, China
[2]Inner Mongolia Key Laboratory of Intelligent Diagnosis and Control of Mechatronic System, Baotou 014010, China
[3]Cuyo University, San Carlos de Bariloche, Argentina

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Abstract: Rolling bearing is the key part of mechanical system. Accurate prediction of bearing life can reduce maintenance costs, improve availability, and prevent catastrophic consequences, aiming at solving the problem of the nonlinear, random and small sample problems faced by rolling bearings in actual operating conditions. In this work, the nonlinear Wiener process with random effect and unbiased estimation of unknown parameters is used to predict the remaining useful life of rolling bearings. Firstly, random effects and nonlinear parameters are added to the traditional Wiener process, and a parameter unbiased estimation method is used to estimate the positional parameters of the constructed Wiener model. Finally, the model is validated using a common set of bearing datasets. Experimental results show that compared with the traditional maximum likelihood function estimation method, the parameter unbiased estimation method can effectively improve the accuracy and stability of the parameter estimation results. The model has a good fitting effect, which can accurately predict the remaining useful life of rolling bearing.

Keywords: parameter estimation; remaining useful life; rolling bearing; Wiener process

I. INTRODUCTION

In modern production machinery, rolling bearing is an important bearing moving part in mechanical systems and plays an important role in the safe and stable operation of mechanical equipment. Carrying out research on the remaining useful life (RUL), prediction of rolling bearing and monitoring the health status of rolling bearing in real time has significant practical application value and important research significance for ensuring the safe and reliable operation of equipment and improving the maintenance efficiency of equipment [1]. Moreover, the high-precision life prediction results can reduce the downtime of mechanical products and provide a theoretical basis for the research of preventive maintenance strategies for mechanical products [2].

Limited by the requirement of comprehensive and complete physical failure mechanism, it is difficult to realize the life prediction method of mechanical products based on the physical failure model. The data-driven life prediction method is more economical and feasible [3]. In data-driven life prediction methods, intelligent algorithms represented by deep learning are similar to "black boxes," with unclear mathematical properties and physical meanings, and poor interpretability. The accuracy of life prediction result of intelligent algorithm is directly related to the sample size and sample quality used for model training, especially for some highly reliable and long-life equipment [4]. Data-driven methods based on mathematical statistics include stochastic processes such as Wiener process [5], gamma process [6], and inverse Gaussian process [7] and have been proven appropriate to model degradation with inherent random effect. Stochastic processes have great mathematical properties for characterizing reversible degradation signals [8] and are used in a wide range of applications, including electronic devices [9,10], mechanical structures [11], and electromechanical systems [12]. Stochastic process model can offer trade-offs between complexity, cost, precision, and applicability, since it possesses favorable mathematical characteristics, reasonable physical interpretations, and wide applicability. Therefore, the data-driven method based on mathematical statistics is the key method to solve the problem of rolling bearing life prediction.

With the development of modern industry, the comprehensiveness of equipment and the variability of the working environment are getting higher and higher. The failure mode of equipment has diversified, and the performance of equipment has correspondingly appeared random and nonlinear degradation. As one of the statistically driven methods of stochastic processes, the Wiener model has the advantages of simple structure and easy adaptive expansion [13]. It can use random feature and nonlinear function terms to express random effects and nonlinearities in the degradation process of mechanical equipment. In 1993, Doksum and Hoyland first used the Wiener model to describe the degradation process of devices [14]. TSAI *et al.* constructed the device life distribution of the Wiener process based on the principle that the device first reached the threshold as the principle of the remaining service life [15]. Yan *et al.* combined the Wiener process with Bayesian method to

Corresponding author: Chao Zhang (e-mail: zhanghero@imust.edu.cn).

enable online updates of the product degradation process using historical O&M data from the device and real-time observation data [16]. Zhao *et al.* consider that the basic linear Wiener model has a large error in describing the nonlinear degradation process of the aero engine [17]. The exhaust temperature margin is used as a performance index to construct the nonlinear drift Wiener process to improve the accuracy of life prediction. Liu *et al.* constructed a Wiener model with a random error term, taking into account individual differences in devices as well as measurement errors [18]. Li *et al*. successfully used the Wiener model to describe the degradation process of rolling bearing by using the parametric maximum likelihood estimation method [19]. Wang *et al*. verified that the drift coefficient of Wiener model correlates with the diffusion coefficient and combined the Kalman filtering method with the parametric maximum likelihood estimation method to achieve bearing life prediction [20]. The above methods have approached the construction process of the Wiener model, but when estimating the parameters of the model, most of them use the maximum likelihood estimation method. Although the maximum likelihood estimation method is relatively simple and convenient to implement for the analytical form of parameter estimation, but the accuracy of the parameter estimation result is still directly related to the sample size. Similar to the high reliability mechanical equipment of rolling bearings, the amount of fault data that can be used for life prediction research is not much. Small sample datasets can have an impact on the accuracy of bearing life predictions.

Therefore, in order to further improve the accuracy of the Wiener model in the application of equipment residual service life prediction, a rolling bearing RUL prediction method combined with the stochastic nonlinear Wiener model and the parameter unbiased estimation method is proposed. The method takes into account the randomness, nonlinearity, and small sample characteristics of the bearing in the degradation process and considers the effect of random effects in the nonlinear Wiener process. In addition, an unbiased parameter estimation method suitable for Wiener model is proposed to reduce the influence of small sample characteristics on the accuracy of life prediction results and then realize the RUL prediction of rolling bearing.

II. NONLINEAR WIENER MODEL CONSTRUCTION CONSIDERING RANDOM EFFECTS

Wiener process, also named Brown operating process, is well suited to describe monotonic or non-monotonic device performance degradation process. It is widely used in mathematical statistical model [21], where the traditional Wiener process can be defined as a continuous-time stochastic process $\{X(t), t \geq 0\}$.

where $x(0) = 0$, $X(t)$ is the amount of change at the moment of t, has a stationary, independent increment independent of the beginning of time. For each $t > 0$, $X(t)$ follows a normal distribution with a mean of 0 and a variance of $\sigma^2 t$. If $\sigma = 1$, the process is called the standard Wiener process. It is always a random motion with the mean of zero near the beginning. This phenomenon is obviously inconsistent with the actual situation. Therefore, considering the number of drift systems λ and the diffusion coefficient β that characterize the degradation rate of the device, there is a tendency for the standard Wiener process to move away from the initial position, where the drift coefficient λ is linear, and the Wiener process can be expressed as Eq. (1):

$$X(t) = X(0) + \lambda t + \beta B(t) \tag{1}$$

where $X(0)$ is the initial variation and $B(t)$ is the standard Brownian motion. For the unary Wiener process expressed in Eq. (1), if the failure threshold of the device is set to $h(h > 0)$, the RUL of the device is defined as the time when the amount of equipment degradation first reaches or exceeds h during its performance degradation process, which is expressed as Eq. (2):

$$T = \inf\{t : X(t) \geq h | X(0) \leq h\} \tag{2}$$

In the formula, $h(h > 0)$ is the failure threshold of the equipment, which is usually formulated by relevant industry standards or expert experience. Correspondingly, the RUL L_k at time point t_k can be expressed as Eq. (3):

$$L_k = \inf\{l_k : X(t_k + l_k) \geq h | X(t_k) \leq h\} \tag{3}$$

where l_k represents the current time point t_k is the time distance from the device failure. Then, the device life distribution and probability density functions that obey the Wiener process can be expressed as Eq. (4):

$$\begin{aligned} F(t) &= \Phi\left(\frac{\lambda t - h}{\beta\sqrt{t}}\right) + \exp\left(\frac{2\lambda h}{\beta^2}\right)\Phi\left(\frac{-\lambda t - h}{\beta\sqrt{t}}\right) \\ f(t) &= \frac{1}{\sqrt{2\pi\beta^2 t^3}}\exp\left(-\frac{(h-\lambda t)^2}{2\beta^2 t}\right) \end{aligned} \tag{4}$$

where the probability density function $f(t)$ can describe the possibility of the output value of random variable t near a certain value point. The cumulative distribution function $F(t)$ is an integral representation of $f(t)$, which can fully describe the probability distribution of random variable t.

However, for bearings, even the same type of bearing produced in the same batch cannot guarantee that the degradation process is completely consistent, and the linear drift coefficient cannot meet the different randomness needs of individual bearings. Therefore, a drift coefficient λ with random effects is necessary for the Wiener process, which can be expressed as Eq. (5):

$$X(t) = X(0) + \lambda t + \beta B(t), \lambda \sim N(\mu, \sigma^2) \tag{5}$$

where drift coefficient λ follows the normal distribution with mean μ and variance σ^2. Correspondingly, bearing life distribution with random effects and probability density function can be expressed as Eq. (6):

$$\begin{aligned} F(t) &= \Phi\left(\frac{\mu t - h}{\sqrt{\sigma^2 t^2 + \beta^2 t}}\right) + \exp\left(\frac{2\mu h}{\beta^2} + \frac{2\sigma^2 h^2}{\beta^4}\right) \\ &\quad \times \Phi\left(\frac{2\sigma^2 h t + \beta^2(\mu t + h)}{\beta^2\sqrt{\sigma^2 t^2 + \beta^2 t}}\right) \\ f(t) &= \sqrt{\frac{h^2}{2\pi t^3(\sigma^2 t + \beta^2)}}\exp\left(-\frac{(h-\mu t)^2}{2t(\sigma^2 t + \beta^2)}\right) \end{aligned} \tag{6}$$

In addition to the randomness of the actual degradation process of the bearing, it also has the nonlinear characteristics common to mechanical equipment products. In order to further improve the similarity between the Wiener process and the bearing degradation process, the nonlinear

Wiener process can be expressed as Eq. (7):

$$X(t) = X(0) + \lambda\Lambda(t;\theta) + \beta B(t) \tag{7}$$

where $\lambda \sim N(\mu_\lambda,\sigma_\lambda^2)$, $\Lambda(t;\theta)$ is a monotonous continuous nonlinear function, where θ represents the nonlinearity of the degenerative process. Obviously, if $\lambda(t;\theta) = \lambda$, the nonlinear Wiener process will become the traditional linear drift Wiener process, which is general.

III. ESTIMATION UNKNOWN PARAMETERS OF THE DEGRADATION PROCESS

A. PARAMETER ESTIMATION OF TRADITIONAL LINEAR WIENER PROCESS

As a mathematical statistics method in the field of life prediction, Wiener process has its advantages of simple structure and strong explanatory ability. In order to show the advantages of the unknown parameter unbiased estimation method used in this paper, firstly, we introduce the maximum likelihood function method commonly used in statistical model parameter estimation briefly. More commonly used linear Wiener process expression is shown in Eq. (1), whose prior model parameter is $\Theta = \{\mu,\sigma^2,\beta^2\}$.

To use the maximum likelihood function to estimate the unknown parameters of the model, Researcher needs to build the log likelihood function of the degenerate model. The full life cycle data of n same design bearings have been completed before the RUL prediction of the rolling bearings is made. The bearing degradation data measured at time $t_{i,j}$ for the ith bearing can be expressed as Eq. (8):

$$x_{i,j} = \lambda_i t_{i,j} + \beta B(t_{i,j}), i \in [1,n], j \in [1,m_i] \tag{8}$$

where λ_i is the average drift coefficient of the ith bearing and m_i is the failure time point of the ith bearing. Correspondingly, the test time vector for the ith bearing and its performance degradation data vector can be expressed as Eq. (9):

$$x_i = (x_{i,1},x_{i,2},\ldots,x_{i,m_i})^T \quad t_i = (t_{i,1},t_{i,2},\ldots,t_{i,m_i})^T \tag{9}$$

Then, the performance degradation data of n training bearings can be expressed as $x = (x_1,x_2,\ldots x_n)^T$ where x_i follows a multidimensional normal distribution with the mean μt and the variance Σ', where $\Sigma' = \sigma^2 tt^T + \beta^2\gamma'$. The difference in degradation data for the same bearing at different points in time is expressed as Eq. (10):

$$\Delta x_{i,j} = x_{i,j} - x_{i,j-1} = \lambda\Delta t_j + \beta B(\Delta t_{i,j}) \tag{10}$$

where $\Delta t_{i,j} = t_{i,j} - t_{i,j-1}, j \in [1,m_i]$. According to Eq. (7), it can be obtained as Eq. (11):

$$\begin{aligned} \Delta x_i &= (\Delta x_{i,1},\Delta x_{i,2},\ldots,\Delta x_{i,m_i})^T \\ \Delta t_i &= (\Delta t_{i,1},\Delta t_{i,2},\ldots,\Delta t_{i,m_i})^T \\ \Delta x &= (\Delta x_1^T,\Delta x_2^T,\ldots,\Delta x_n^T)^T \end{aligned} \tag{11}$$

Then, Δx_i follows a multidimensional normal distribution with the mean $\mu\Delta t_i$ and the variance Σ_i, where $\Sigma_i = \sigma^2\Delta t_i \Delta t_i{}^T + \beta^2\gamma_i$, $\gamma_i = diag(\Delta t_{i,1},\Delta t_{i,2},\ldots,\Delta t_{i,m_i})$. The log likelihood function of argument $\Theta = \{\mu,\sigma^2,\beta^2\}$ of the prior model of the linear Wiener process can be expressed as Eq. (12):

$$\begin{aligned} \ln L(\Theta|X) =& -\frac{1}{2}\ln 2\pi\sum_{i=1}^{n} m_i - \frac{n}{2}\ln|\Sigma_i| \\ & -\frac{1}{2}\sum_{i=1}^{n}(\Delta x_i - \mu\Delta t_{i,m_i})^T\Sigma_i^{-1}(\Delta x_i - \mu\Delta t_{i,j}) \\ =& -\frac{1}{2}\ln 2\pi\sum_{i=1}^{n} m_i - \frac{n}{2}\ln(\beta^2)^{m_i-1} \\ & -\frac{n}{2}\ln(\beta^2 + \sigma^2 t_{i,m_i}) - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m_i}\ln\Delta t_{i,j} \\ & -\frac{1}{\beta^2}\left[\frac{1}{2}\sum_{i=1}^{n}\Delta x_i^T\gamma_i^{-1}\Delta x_i - \frac{1}{2}\sum_{i=1}^{n}\frac{x_{i,m_i}^2}{t_{i,m_i}}\right] \\ & -\frac{1}{2}\sum_{i=1}^{n}\frac{t_{i,m_i}}{\beta^2 + \sigma^2 t_{i,m_i}}\left(\mu - \frac{x_{i,m_i}}{t_{i,m_i}}\right)^2 \end{aligned} \tag{12}$$

Then, the log likelihood function shown in Eq. (10) takes the first-order partial derivative for the unknown parameter μ,σ^2, respectively, and the result is shown in the following equation:

$$\begin{aligned} \frac{\partial\ln L(\Theta|X)}{\partial\mu} &= -\frac{1}{\beta^2 + \sigma^2 t_{i,m_i}}\left(\sum_{i=1}^{n} x_{i,m_i} - n\mu t_{i,m_i}\right) \\ \frac{\partial\ln L(\Theta|X)}{\partial\sigma^2} &= -\frac{n t_{i,m_i}}{2(\beta^2 + \sigma^2 t_{i,m_i})} \\ & \quad + \frac{t_{i,m_i}^2}{2(\beta^2 + \sigma^2 t_{i,m_i})}\sum_{i=1}^{n}\left(\mu - \frac{x_{i,m_i}}{t_{i,m_i}}\right)^2 \end{aligned} \tag{13}$$

Each term in Eq. (11) is equal to 0 respectively, and the result is Eq. (14):

$$\hat{\mu} = \frac{1}{t_{i,m_i}}\sum_{i=1}^{n} x_{i,m_i} \quad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{\mu} - \frac{x_{i,m_i}}{t_{i,m_i}}\right)^2 - \frac{\hat{\beta}^2}{t_{i,m_i}} \tag{14}$$

From Eq. (12), it is possible to obtain a maximum likelihood estimate of parameter μ, but the estimate of σ^2 is related to β, so Eq. (12) needs to be substituted into Eq. (10), and it can be obtained as Eq. (15):

$$\begin{aligned} \ln L(\beta^2|X) =& -\frac{1}{2}\ln 2\pi\sum_{i=1}^{n} m_i - \frac{n}{2}\ln(\beta^2)^{m_i-1} \\ & -\frac{n}{2}\ln\left(\frac{t_{i,m_i}}{n}\sum_{i=1}^{n}\left(\hat{\mu} - \frac{x_{i,m_i}}{t_{i,m_i}}\right)^2\right) - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{m_i}\ln\Delta t_{i,j} \\ & -\frac{1}{\beta^2}\left[\frac{1}{2}\sum_{i=1}^{n}\Delta x_i^T\gamma_i^{-1}\Delta x_i - \frac{1}{2}\sum_{i=1}^{n}\frac{x_{i,m_i}^2}{t_{i,m_i}}\right] - \frac{1}{2}n \end{aligned} \tag{15}$$

By equaling the first-order partial derivative of Eq. (13) to parameter β^2 to 0, we can obtain a maximum likelihood estimate for parameter β^2 as Eq. (16):

$$\hat{\beta}^2 = \frac{1}{n(m_i - 1)}\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{(\Delta x_{i,j} - \frac{x_{i,m_i}}{t_{i,m_i}}\Delta t_{i,j})^2}{\Delta t_{i,j}} \tag{16}$$

Once the estimated value of parameter β^2 is known, the maximum likelihood estimate of parameter σ^2 is as Eq. (17):

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{\mu} - \frac{x_{i,m_i}}{t_{i,m_i}}\right)^2 - \frac{1}{n(m_i-1)t_{i,m_i}}\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{(\Delta x_{i,j} - \frac{x_{i,m_i}}{t_{i,m_i}}\Delta t_{i,j})^2}{\Delta t_{i,j}} \quad (17)$$

The parameter unbiased estimation of the traditional linear Wiener process is summarized as Eq. (18):

$$\begin{aligned}\dot{\mu} &= \frac{1}{n}\sum_{i=1}^{n}\frac{x_{i,m_i}}{t_{i,m_i}}\\ \dot{\sigma}^2 &= \frac{1}{n-1}\sum_{i=1}^{n}\left(\dot{\mu} - \frac{x_{i,m_i}}{t_{i,m_i}}\right)^2 - \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{(\Delta x_{i,j} - \frac{x_{i,m_i}}{t_{i,m_i}}\Delta t_{i,j})^2}{(m_i-1)t_{i,m_i}\Delta t_{i,j}}\\ \dot{\beta}^2 &= \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{(\Delta x_{i,j} - \frac{x_{i,m_i}}{t_{i,m_i}}\Delta t_{i,j})^2}{(m_i-1)\Delta t_{i,j}}\end{aligned} \quad (18)$$

Through the most direct comparison between Eqs. (12), (14), (15), and (16), it can be seen that the unbiased estimate of parameter σ is also less than zero, because the first term of the unbiased estimate of σ is smaller, the corresponding probability value will be lower, which can improve the accuracy of the parameter estimate to a certain extent. In the overall parameter estimation expression, it can be clearly seen that the smaller the n value of the bearing life sample size, the greater the difference between the result accuracy of the unbiased estimation of the parameters and the accuracy of the result of the biased estimate. So compared with the traditional parameter great likelihood estimation method, the unbiased estimation method is more suitable for bearing mechanical products with a limited sample size.

B. UNBIASED PARAMETER ESTIMATION OF NONLINEAR WIENER PROCESSES

Compared with the traditional linear Wiener process, the prior model parameter of nonlinear Wiener process Eq. (5) is $\Theta = \{\mu,\sigma^2,\beta^2,\theta\}$, and for the full life cycle data of n bearings, the bearing degradation data measured by the ith bearing at the time $t_{i,j}$ of its degradation process can be expressed as Eq. (19):

$$x_{i,j} = \lambda_i\Lambda(t_{i,j};\theta) + \beta B(t_{i,j}), i \in [1,n], j \in [1,m_i] \quad (19)$$

Similarly, the degradation data difference of the same bearing at different points in time is expressed as Eq. (20):

$$\Delta x_{i,j} = x_{i,j} - x_{i,j-1} = \lambda\Delta\delta_{i,j} + \beta B(\Delta t_{i,j}) \quad (20)$$

where $\Delta\delta_{i,j} = \delta_{i,j} - \delta_{i,j-1} = \Lambda(t_{i,j};\theta) - \Lambda(t_{i,j-1};\theta)$, $\Delta t_{i,j} = t_{i,j} - t_{i,j-1}, j \in [1,m_i]$. Then, the performance degradation data vector of the ith bearing, the test time vector, etc. can be expressed as Eq. (21):

$$\begin{aligned}\Delta x_i &= (\Delta x_{i,1},\Delta x_{i,2},\ldots,\Delta x_{i,m_i})^T\\ \Delta t_i &= (\Delta t_{i,1},\Delta t_{i,2},\ldots,\Delta t_{i,m_i})^T\\ \Delta x &= (\Delta x_1^T,\Delta x_2^T,\ldots,\Delta x_n^T)^T\\ \Delta\delta_i &= (\Delta\delta_{i,1},\Delta\delta_{i,2},\ldots,\Delta\delta_{i,m_i})^T\end{aligned} \quad (21)$$

Then, Δx_i follows a multidimensional normal distribution with the mean $\mu\Delta\delta_i$ and the variance $\dot{\Sigma}_i$, where $\dot{\Sigma}_i = \sigma^2\Delta\delta_i\Delta\delta_i^T + \beta^2\dot{\gamma}_i$, $\dot{\gamma}_i = diag(\Delta t_{i,1},\Delta t_{i,2},\ldots,\Delta t_{i,m_i})$. The log likelihood function of prior model parameter $\Theta = \{\mu,\sigma^2,\beta^2,\theta\}$ of a nonlinear Wiener process can be expressed as Eq. (22):

$$\ln L(\Theta|X) = -\frac{1}{2}\ln 2\pi\sum_{i=1}^{n}m_i - \frac{n}{2}\ln|\dot{\Sigma}_i| - \frac{1}{2}\sum_{i=1}^{n}(\Delta x_i - \mu\Delta\delta_i)^T\dot{\Sigma}_i^{-1}(\Delta x_i - \mu\Delta\delta_i) \quad (22)$$

Similar to Section 2.1, the estimate of the parameter is found using the method of maximizing the value of Eq. (20) as Eq. (23):

$$\begin{aligned}\dot{\mu}(\dot{\theta}) &= \frac{1}{n}\sum_{i=1}^{n}\frac{\Delta x_i^T\gamma_i^{-1}\Delta\delta_i}{\Delta\delta_i^T\gamma_i^{-1}\Delta\delta_i}\\ \dot{\sigma}^2(\dot{\theta}) &= \frac{1}{n}\sum_{i=1}^{n}\left(\dot{\mu} - \sum_{i=1}^{n}\frac{\Delta x_i^T\gamma_i^{-1}\Delta\delta_i}{\Delta\delta_i^T\gamma_i^{-1}\Delta\delta_i}\right)^2 - \frac{1}{n}\sum_{i=1}^{n}\frac{\left(\Delta x_i - \frac{\Delta x_i^T\gamma_i^{-1}\Delta\delta_i}{\Delta\delta_i^T\gamma_i^{-1}\Delta\delta_i}\Delta\delta_i\right)^T\gamma_i^{-1}\left(\Delta x_i - \frac{\Delta x_i^T\gamma_i^{-1}\Delta\delta_i}{\Delta\delta_i^T\gamma_i^{-1}\Delta\delta_i}\Delta\delta_i\right)}{(m_i-1)\Delta\delta_i^T\gamma_i^{-1}\Delta\delta_i}\\ \dot{\beta}^2(\dot{\theta}) &= \frac{1}{n}\sum_{i=1}^{n}\frac{\left(\Delta x_i - \frac{\Delta x_i^T\gamma_i^{-1}\Delta\delta_i}{\Delta\delta_i^T\gamma_i^{-1}\Delta\delta_i}\Delta\delta_i\right)^T\gamma_i^{-1}\left(\Delta x_i - \frac{\Delta x_i^T\gamma_i^{-1}\Delta\delta_i}{\Delta\delta_i^T\gamma_i^{-1}\Delta\delta_i}\Delta\delta_i\right)}{(m_i-1)}\end{aligned} \quad (23)$$

As can be seen from Eq. (21), in the parametric estimation process of the nonlinear Wiener process, the value of parametric μ,σ^2,β^2 is directly related to the estimation of θ. If the estimation of parameter θ satisfies $\dot{\theta} = \theta$, then the parameter estimation expected value for the nonlinear Wiener process is Eq. (24):

$$\begin{aligned}E(\dot{\mu}) &= \mu\\ E(\dot{\sigma}^2) &= \frac{n-1}{n}\sigma^2 - \frac{\beta^2}{n\Delta\delta_i^T\gamma_i^{-1}\Delta\delta_i}\\ E(\dot{\beta}^2) &= \beta^2\end{aligned} \quad (24)$$

It is clear from Eq. (22) that the estimate of parameter σ is biased, so for a nonlinear Wiener process, the unbiased estimation expression for its parameter value is Eq. (25):

$$\begin{aligned}\dot{\mu}(\dot{\theta}) &= \frac{1}{n}\sum_{i=1}^{n}\frac{\Delta x_i^T\gamma_i^{-1}\Delta\delta_i}{\Delta\delta_i^T\gamma_i^{-1}\Delta\delta_i}\\ \dot{\sigma}^2(\dot{\theta}) &= \frac{1}{n-1}\sum_{i=1}^{n}\left(\dot{\mu} - \sum_{i=1}^{n}\frac{\Delta x_i^T\gamma_i^{-1}\Delta\delta_i}{\Delta\delta_i^T\gamma_i^{-1}\Delta\delta_i}\right)^2 - \frac{1}{n}\sum_{i=1}^{n}\frac{\left(\Delta x_i - \frac{\Delta x_i^T\gamma_i^{-1}\Delta\delta_i}{\Delta\delta_i^T\gamma_i^{-1}\Delta\delta_i}\Delta\delta_i\right)^T\gamma_i^{-1}\left(\Delta x_i - \frac{\Delta x_i^T\gamma_i^{-1}\Delta\delta_i}{\Delta\delta_i^T\gamma_i^{-1}\Delta\delta_i}\Delta\delta_i\right)}{(m_i-1)\Delta\delta_i^T\gamma_i^{-1}\Delta\delta_i}\\ \dot{\beta}^2(\dot{\theta}) &= \frac{1}{n}\sum_{i=1}^{n}\frac{\left(\Delta x_i - \frac{\Delta x_i^T\gamma_i^{-1}\Delta\delta_i}{\Delta\delta_i^T\gamma_i^{-1}\Delta\delta_i}\Delta\delta_i\right)^T\gamma_i^{-1}\left(\Delta x_i - \frac{\Delta x_i^T\gamma_i^{-1}\Delta\delta_i}{\Delta\delta_i^T\gamma_i^{-1}\Delta\delta_i}\Delta\delta_i\right)}{(m_i-1)}\end{aligned} \quad (25)$$

IV. LIFE PREDICTION MODEL FOR ROLLING BEARINGS

Rolling bearings are widely used in the field of mechanical engineering. Their operating conditions are usually complex, and the corresponding bearing performance degradation process is difficult to generalize [22]. For the general nonlinear Wiener process Eq. (5), the stochastic degradation process $\{X(t), t \geq 0\}$ of the bearing can be defined as Eq. (26):

$$X(t) = X(0) + \int_0^t \lambda(t;\theta)dt + \beta B(t) \tag{26}$$

where $\lambda(t;\theta)$ is the drift coefficient. In order to make the summed up bearing probability density function closer to the actual working conditions, scholars have conducted in-depth research; usually, the nonlinear function $\lambda(t;\theta)$ is expressed in the form of a power function or an exponential function. According to the literature [23], the power function $abt^{b-1}, a\sim N(\mu_a,\sigma_a^2)$ is selected to characterize the drift function, where parameter a is a random parameter that represents the individual difference of the bearing, b is the commonality of bearing degradation, and the unknown parameter θ in the drift parameter can be replaced by (a,b). Then, the probability density function for the degradation model of the bearing to reach the failure threshold $h(h > 0)$ for the first time is Eq. (27):

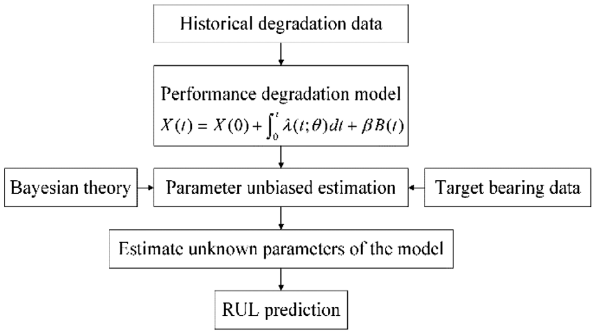

Fig. 1. Flowchart of the RUL prediction method.

$$f_b(t) \cong \frac{h - (t^b - bt^b)\frac{h\sigma_a^2 t^{(b-1)} + \mu_a \beta^2}{\sigma_a^2 t^{(2b-1)} + \beta^2}}{\sqrt{2\pi t^3(\sigma_a^2 t^{(2b-1)} + \beta^2)}} \exp\left(-\frac{(h - \mu_a t^b)^2}{2t(\sigma_a^2 t^{(2b-1)} + \beta^2)}\right) \tag{27}$$

If the performance degradation of the rolling bearing at time t_k is $X(t_k)$, according to the explanation of the RUL in Eq. (3), it can be obtained that the probability density function of the RUL of the bearing at time t_k is

$$\begin{aligned} f_b(l_k) \cong & \frac{h - X(t_k) - ((l_k + t_k)^b - t_k^b - bl_k(l_k + t_k)^{b-1})\frac{(h-X(t_k))\sigma_a^2((l_k+t_k)^b - t_k^b) + \mu_a\beta^2 l_k}{\sigma_a^2((l_k+t_k)^b - t_k^b)^2 + \beta^2 l_k}}{\sqrt{2\pi {l_k}^2(\sigma_a^2((l_k + t_k)^b - t_k^b)^2 + \beta^2 l_k)}} \\ & \times \exp\left(-\frac{((h - X(t_k)) - \mu_a((l_k + t_k)^b - t_k^b))^2}{2(\sigma_a^2((l_k + t_k)^b - t_k^b)^2 + \beta^2 l_k)}\right) \end{aligned} \tag{28}$$

In the unknown parameter $\Theta = \{\mu,\sigma,\beta,b\}$of the degradation model, β,b can represent the inherent randomness of individual rolling bearings and does not change in the process of bearing complete performance degradation. μ,σ is used to represent the differences between individual rolling bearings, and the parameters will constantly change with the operation of bearings.

Therefore, the historical training data of bearings can be used to estimate the initial value of model parameters and determine the fixed parameters. The random parameters can be determined by using target bearing data and Bayesian theory. The relevant introduction of Bayesian theory can be referred to the literature [24].

When building life prediction models based on historical degradation data from rolling bearings, different combinations of variables mean different models. Therefore, the evaluation of the model is very important [25]. Nonlinear Wiener model of bearing can be evaluated from two aspects: maximizing the likelihood function of the model and minimizing the number of unknown parameters. The general understanding is that larger the value of the model likelihood function, the better the fitting effect of the model. But it will lead to an increase in the number of unknown parameters in the model, increase the complexity of the model and the cost of calculation time, and even lead to overfitting of the model. Therefore, the Akaike Information Criterion (AIC) and the mean-square error (MSE) are introduced, where the function expression of AIC is Eq. (29):

$$AIC = -2\ln(L) + 2k \tag{29}$$

where L is the likelihood function of the model and k is the unknown number of parameters of the model that participate in the parameter estimation. Compared with the Bayesian Information Criterion (BIC), the AIC criterion is more suitable for life prediction model with limited sample size, and its penalty factor is independent of the sample size, which can take into account the degree of fit of the model and the number of unknown parameters. The smaller the AIC value, the better the fit of the model. The function expression of the MSE is Eq. (30):

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{m_i}\sum_{j-1}^{m_i}(\hat{F}(t_{i,j},\Theta) - \widetilde{F}(t_{i,j}))^2 \tag{30}$$

where $\hat{F}(t_{i,j},\Theta)$ is an estimate of the cumulative probability function in time $t_{i,j}$ with respect to Θ, while $\widetilde{F}(t_{i,j})$ is the empirical value of the cumulative probability function at time $t_{i,j}$. MSE can be used to detect the precision of model training results. The flow chart of this method is shown in Fig. 1.

V. CASE STUDIES

In order to further illustrate the feasibility and effectiveness of the nonlinear Wiener process in the study of rolling bearing RUL, the accelerated bearing degradation data

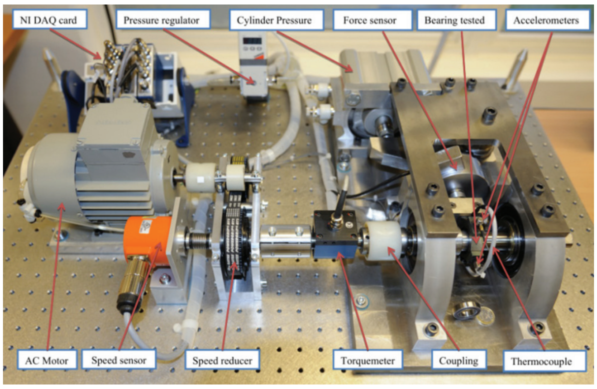

Fig. 2. PRONOSTIA test platform.

collected by the PRONOSTIA laboratory was used for example verification. This bearing dataset is a public data set. The parameters related to the specific data can be referred to [26]. The experimental table is shown in Fig. 2.

In this paper, seven datasets in working condition one are selected to verify the parameter evaluation results of the life prediction model and the accuracy of the life prediction results. The third set of bearing data 1_3 in working condition one is used as the target bearing for the RUL prediction, and the remaining six datasets of bearing are used to estimate the unknown parameter values of the degradation model. In order to highlight the unbiased parameter estimation method compared with the traditional maximum likelihood function parameter estimation method, the degradation model of the bearing selects the nonlinear Wiener process with random effects as shown in Eq. (28) and evaluates the degradation model parameters of the bearing using two parameter estimation methods. The estimation results of the parameters and the evaluation function calculation results of the model are shown in Table I.

As can be seen from Table I, for the parameter estimation results of the degradation model, the traditional maximum likelihood function method and the parameter unbiased estimation method have little difference between the estimation results of the unknown parameters μ, b, and β. Especially, the estimation of parameter μ is exactly the same. The large difference in estimation is the unknown parameter σ, and the unbiased estimation method has a larger estimation of parameter σ, although it cannot be completely avoided, but it can reduce the risk of parameter $\sigma < 0$ to a certain extent. The calculation results of AIC and MSE, the evaluation functions of model fitting results, show that the parameter unbiased estimation method can effectively improve the accuracy of the bearing degradation model, reduce the risk of parameters less than zero to a certain extent, and improve the stability of model parameter estimation. In order to verify the effectiveness and superiority of the proposed method, the nonlinear Bayesian method is set as the comparison method. The detailed

Table I. Results of fitting bearing degradation data

	Unknown Parameter					
Parameter Estimation Method	**μ**	**σ**	**b**	**β**	**AIC**	**MSE**
Traditional maximum likelihood function	0.2375	0.0018	0.9420	0.0628	−51.36	0.0036
Parameter unbiased estimation method	0.2375	0.0023	0.9443	0.0627	−52.74	0.0034

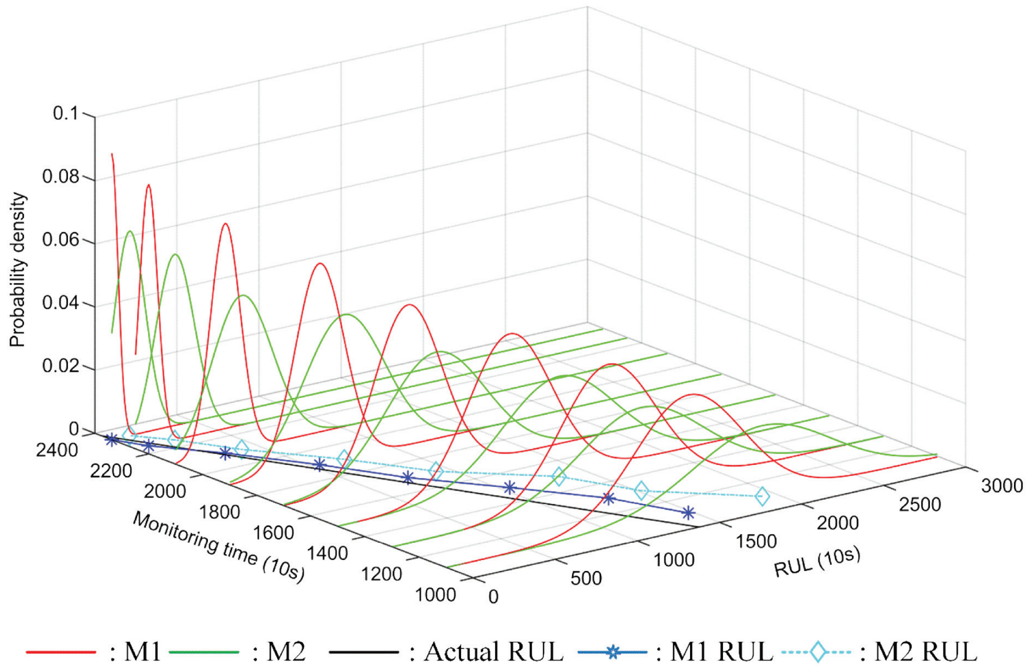

Fig. 3. PDFs of RUL under M1 and M2.

description of nonlinear Bayesian method can be found in reference [27]. For simplicity, the method proposed in this paper is referred to as M1, and the method based on nonlinear Bayesian method is referred to as M2. We plot the RUL distributions of the two methods at some different time points, as shown in Fig. 3.

It can be seen from Fig. 3 that the RUL distribution calculated by both methods can cover the actual RUL, although the true RUL of the bearing remains mostly within the range of the probability density function, even at the beginning of the experiment. However, the function curve is flat and less precise. As the monitoring time of the bearing continues to increase, the corresponding amount of performance degradation data also increases, and the probability density function of the bearing becomes steeper, gradually converging near the true remaining life, and the prediction accuracy also becomes higher. Compared with the nonlinear Bayes method, the RUL distribution of the proposed method is more concentrated on the actual RUL, and the PDF distribution is more slender, which indicates that our method has higher accuracy. The more intuitive life prediction is shown in Fig. 4.

In order to illustrate the estimation accuracy of the life prediction results, Absolute Relative Error (ARE) and Root Mean Square Error (RMSE) were used to evaluate the accuracy of the model, as shown in Figs. 5 and 6. For RUL prediction, the proposed method is obviously superior to the comparison method. The method proposed in this paper has small ARE and RMSE in most of the whole prediction stage, which shows the superiority of the method proposed in this paper.

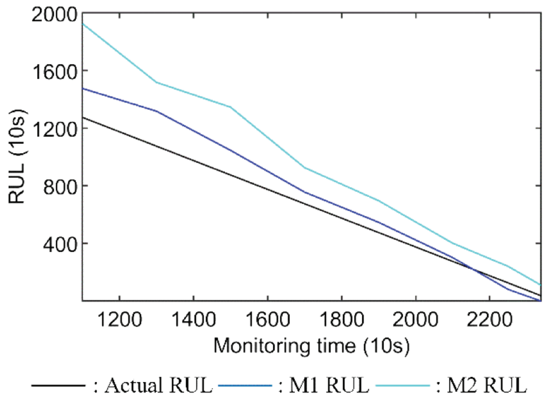

Fig. 4. Predicted life of the bearing.

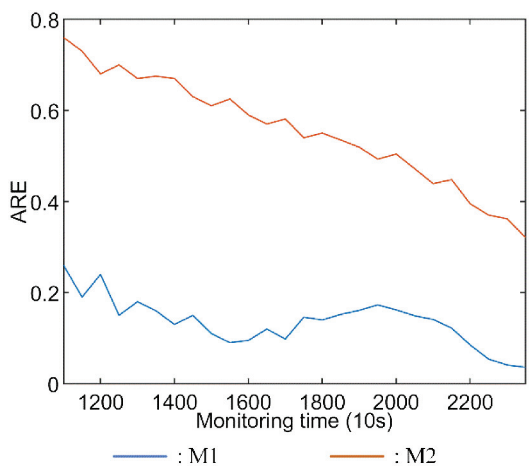

Fig. 5. The ARE by M1 and M2.

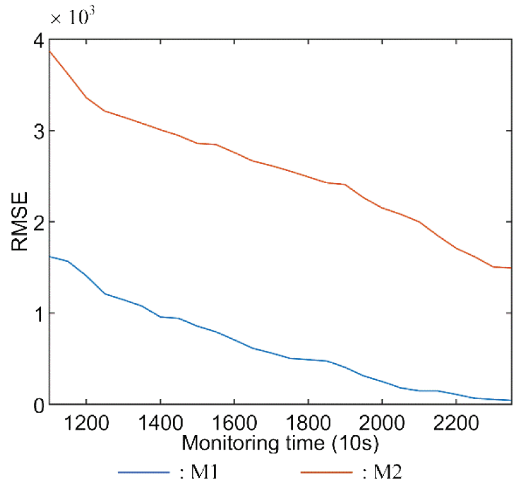

Fig. 6. The RMSE by M1 and M2.

VI. CONCLUSION

On the basis of the traditional Wiener process, random effect and nonlinear feature are added, and the life prediction model is built closer to the actual operating conditions of rolling bearing. In terms of parameter estimation, this paper uses the parameter unbiased estimation method to estimate the unknown parameters of the model, which can effectively reduce the risk of parameters less than zero, and improve the accuracy and stability of the life prediction model. Through the example analysis of the RUL prediction of the PRONOSTIA bearing test platform, the model fitting effect and performance of the thesis method are good, and the performance degradation process of rolling bearings can be effectively described, and the accuracy is high.

Future work will focus on verifying the application of the proposed life prediction method to other mechanical components, as well as generalizations in other data-driven models.

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CONFLICT OF INTEREST STATEMENT

The authors declare no conflicts of interest.

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