

# **A hybrid method for mesh stiffness modelling in the presence of structural defects using the finite element method and local slope Approach**

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## **Abstract**

This work proposes an enhanced hybrid formulation for evaluating the Time-Varying Mesh Stiffness (TVMS) of helical gears in the presence of structural defects. While traditional compliance models often rely on computationally intensive full-scale numerical simulations or oversimplified analytical slicing, this paper introduces a novel extension of the Hybrid Cooley's Approach (HCA) coupled with the Local Slope Method, specifically adapted for helical tooth geometries. The main novelty lies in integrating localized numerical compliance corrections directly into an analytical multi-slice framework, enabling the simultaneous tracking of non-uniform tooth spalling and crack propagation profiles with minimal computational cost. To verify the proposed hybrid formulation and capture localized stress distributions under distinct load configurations, a rigorous three-dimensional Finite Element (FE) model is developed. The comparative outcomes demonstrate that the proposed method effectively bridges the gap between fast analytical routines and high-fidelity numerical solutions, achieving high accuracy in stiffness estimation across varying operational load ranges. This research advances the fundamental

understanding of fault-induced vibration behaviors and establishes a precise, computationally lightweight prognostic tool for early fault detection in industrial power transmissions.

**Keywords:** Time varying Mesh stiffness, Local slope approach, Helical gears, Hybrid Cooley's Approach, Finite element analysis,

|                  |                                  |
|------------------|----------------------------------|
| $K_m$            | TVMS                             |
| $F_m$            | Force variation                  |
| $q_m$            | Gear meshing deflection          |
| $U_g$            | Elastic displacements of gear    |
| $U_p$            | Elastic displacements of piston  |
| $r_{ap}, r_{ag}$ | Radius of pinion / gear          |
| $r_p, r_g$       | Airbrush radius of pinion / gear |
| $\varepsilon$    | Contact ratio                    |

**1.Introduction:**

The structural integrity and operational reliability of mechanical power transmissions represent a cornerstone of modern industrial engineering, especially within high-stakes and safety-critical sectors such as automotive drivetrains, aerospace propulsion, and renewable energy wind turbines, as extensively discussed by G. Blankenship and R. Singh [1] and H. Özgüven [2]. In these demanding fields, helical gears have progressively become the preferred architectural choice over conventional spur gears. Their inherent ability to distribute contact forces smoothly along an oblique line of action, coupled with their high load-carrying capacity and reduced noise emissions, makes them indispensable for high-speed applications—a technical and kinematic superiority well-documented by Ma. H et al. [6] and F. Litvin et al. [9]. However, despite substantial advancements in material science and manufacturing precision, these heavy-duty components remain highly susceptible to structural degradations over long operational life cycles. As noted by P. Velez [4] and F. Chaari et al. [11], sub-surface fatigue cracks at the tooth root fillet and localized surface spalling on the active profiles can silently emerge under cyclic contact stresses. If left undetected, these micro-damages continuously alter the system’s structural compliance and dynamic signature, acting as precursors to severe stress concentrations that can ultimately culminate in catastrophic

structural failure. Consequently, tracking the progressive evolution of these localized faults necessitates the development of high-fidelity physical models that accurately correlate macro-level health degradation with localized compliance variations. Over the past few decades, the dynamic modeling of gear transmissions has shifted from simple lumped-parameter approaches to more advanced, nonlinear finite element methods. While early models were good at capturing the overall torsional behavior, they lacked the detailed spatial resolution needed to identify local stress hotspots and subtle changes in compliance caused by early structural damage. As a result, it's become crucial to combine high-accuracy numerical contact mechanics with streamlined analytical models to achieve both precise structural insights and efficient computation for effective gear health monitoring.

At the core of gear health monitoring, diagnostic modeling, and prognostic frameworks lies a fundamental time-varying parameter: the Time-Varying Mesh Stiffness (TVMS). The continuous fluctuations in TVMS, dictated by the periodic variation in the number of teeth simultaneously in contact and the shifting contact line geometry, are the primary architects of internal dynamic excitation. These fluctuations directly generate dynamic transmission errors, driving the structural vibrations, parametric resonances, and acoustic noise levels that ultimately define a transmission system's global performance. researchers have long recognized that accurately modeling and calculating TVMS, under both pristine and damaged conditions, constitutes the vital first step toward implementing effective early fault detection and reliable predictive maintenance algorithms. Consequently, formulating sensitive mathematical models capable of capturing minute structural deviations and localized fault signatures remain a central objective in state-of-the-art diagnostic algorithms.

Historically, the scientific journey to estimate this fundamental mesh stiffness has heavily relied on analytical and semi-analytical frameworks. For several decades, the potential energy method pioneered by G. Blankenship and R. Singh [1], along with the non-uniform cantilever beam models refined by J. Hou [30] and H. Ma et al. [22][23], has served as the baseline standard. These methods treat each individual gear tooth as a cantilever beam fixed at the root fillet to evaluate bending, shear, and axial compression energies, while integrating empirical formulas to account for Hertzian contact compliance and fillet-foundation elasticity. While these analytical techniques are widely

celebrated for their near-instantaneous computational execution, they inevitably reach their structural limits when applied to complex multi-pair helical contact fronts or when capturing the intricate stress concentrations caused by localized surface defects—a critical limitation highlighted by S. Vijayakar [8] and Y. Li et al. [9][25]. Furthermore, conventional analytical slicing techniques for helical gear profiles typically partition the wide tooth face into independent, two-dimensional slices. By treating these sections as isolated entities, they completely neglect the crucial three-dimensional elastic coupling and shear deformation sharing between adjacent sections of the tooth body. This geometric simplification introduces severe mathematical discrepancies and systematic underestimations when modeling highly non-uniform faults, such as asymmetric tooth root cracks or localized spalling propagation across the face width.

Although traditional potential energy methods and flexible beam models allow for fast calculations, they are based on simplified 2D geometries and assume uniform, isotropic stress. These simplifications miss important details like localized 3D elastic interactions, stress shifts, and edge-loading effects caused by complex faults such as uneven surface spalling or growing root cracks. Because of this, standard analytical models often underestimate changes in mesh compliance, highlighting the need for hybrid approaches that can incorporate local compliance adjustments within detailed multi-slice contact mechanics.

Recently, T. Zhang et al. [36] advanced this field significantly by providing a rigorous framework for validating dynamic excitations under multi-source internal forces, reminding the engineering community that theoretical models must be continuously verified against experimental benchmarks or high-fidelity numerical counterparts. To circumvent the geometric oversimplifications inherent in analytical beam theories, full three-dimensional Finite Element (FE) simulations have been increasingly adopted as the gold standard for contact analysis. Full-scale FE models offer exceptional geometric fidelity, capture comprehensive non-linear contact zone behaviors naturally, and account for complex tooth-body coupling without requiring empirical correction factor assumptions. However, the requirement for ultra-fine mesh resolution within the localized contact zones, combined with highly non-linear iterative boundary contact convergence routines, renders full FE modeling computationally prohibitive. It demands immense CPU processing time and expansive memory resources, making it thoroughly unpractical for extensive parametric

optimization, statistical fault sensitivity analysis, or online, real-time diagnostic applications where processing speed is essential.

Seeking to bridge the operational chasm between "computationally fast but oversimplified" analytical formulations and "highly precise but prohibitively slow" numerical simulations, scholars like S. Baud et al. [14] and S. Feng et al. [16] have championed advanced hybrid strategies. These strategies balance computational economy and localized accuracy by confining heavy non-linear numerical computations within isolated, local tooth contact boundaries, while evaluating the global elastic deformation of the gear structure analytically. To balance accuracy and processing time, recent studies have increasingly turned to hybrid analytical-numerical methods. However, conventional hybrid models often rely on a global average slope across the force-deflection domain, which tends to smooth out local non-linearities under varying operational loads. Integrating a local slope finite-difference scheme into the multi-slice framework makes it possible to capture localized contact behavior with higher sensitivity, striking a practical balance between computation speed and 3D finite element precision.

The proposed study follows this advanced hybrid path, specifically drawing inspiration from the localized deformation framework recently introduced by A-E. Chakroun et al. [35] (2025). While A-E. Chakroun originally tailored this approach for the complex surfaces and localized compliance characteristics of bevel gears, the localized deformation logic is fundamentally adapted, mathematically restructured, and extended here to accommodate the unique helix angles and contact fronts of helical gear geometries. The developed framework is designated as the Hybrid Cooley's Approach (HCA), honoring the foundational analytical legacy of C. Cooley et al. [19]. Relying also on the high validation standards set by T. Zhang [36], this paper delivers an engineering tool that is both highly precise and computationally efficient. This hybrid strategy significantly sharpens the model's sensitivity to predict stiffness degradation under localized defect conditions by integrating the adapted Local Slope Method directly into the multi-slice compliance skeleton. Consequently, this framework establishes a precise, fast, and robust diagnostic routine, adding valuable engineering insights and analytical depth to the existing condition monitoring literature.

The remainder of this paper is structured systematically as follows: Section 2 details the mathematical formulation of the proposed Hybrid Cooley's Approach (HCA) and the geometrical modelling of helical gear teeth considering tooth root crack and surface spalling defects. Section 3 presents the analytical definition and compliance calculation through the adapted Local Slope Method. Section 4 presents the results of the parametric study, showcasing the unique capabilities of the novel hybrid layout and verifying the outcomes against full Finite Element models under various torque configurations. Finally, Section 5 is devoted to concluding remarks, summarizing the key engineering findings and outlining prospective future outlooks.

## **2. Description of the Hybrid Cooley's Approach (HCA) Framework**

The accurate modelling of the TVMS allows an accurate prediction of the dynamic behaviour, vibrations and potential defects of gear systems. The TVMS of helical gears in the presence of defects is assessed here using a hybrid methodology, which will be referred to as HCA. Combining Finite Element (FE) simulations and analytical modelling (a modified Cooley's approach with local slope evaluation) allows this method to balance computational efficiency and accuracy.

### **2.1 Theoretical Foundations of the HCA**

In helical gear sets, contact progresses continuously along an oblique line of action, resulting in complex 3D contact patterns. To model this behavior efficiently, the active face width is divided into multiple narrow coupled slices. Evaluating each slice using the local slope approach allows the model to account for load redistribution, helix angle effects, and local stiffness degradation across the tooth face without massive computational cost.

The TVMS estimation is based on the general approach of the stiffness as the ratio between force and deflection:

$$K_m = \frac{F_m}{q_m} \quad (1)$$

Where  $F_m$  is the meshing force and  $q_m$  is the meshing deflection. The framework for analysis is based on a modified version of Cooley's method [19], which uses the local slope of the nonlinear force–deflection curve to calculate the gear tooth's instantaneous stiffness (see Fig.1). The figure shows the relationship between mesh force  $F_m$  and  $q_m$ . The black curve illustrates the non-linear behaviour between force and deflection that is the result of contact between teeth. The finite-

difference estimation can be made through either a local or an average slope. The average slope (blue line) simplifies the problem by assigning an equivalent overall stiffness to the system, which is convenient in simplified analytical models. In addition, the average slope has two major limitations: it does not take into account local variations in stiffness, and it is not representative in a dynamic analysis. The second method is based on the local slope (red line), defined as the tangent to the curve at the corresponding measurement point. This finite difference approach is relatively accurate to estimate the actual value of the force at a corresponding deflection, as it accurately reflects the non-linearity of the engagement and allows for greater precision in dynamic calculations (vibration, noise and stability). Finally, the parameter  $\epsilon$  represents the threshold deflection below which the engagement is not effective and is used to identify the start of the contact area.

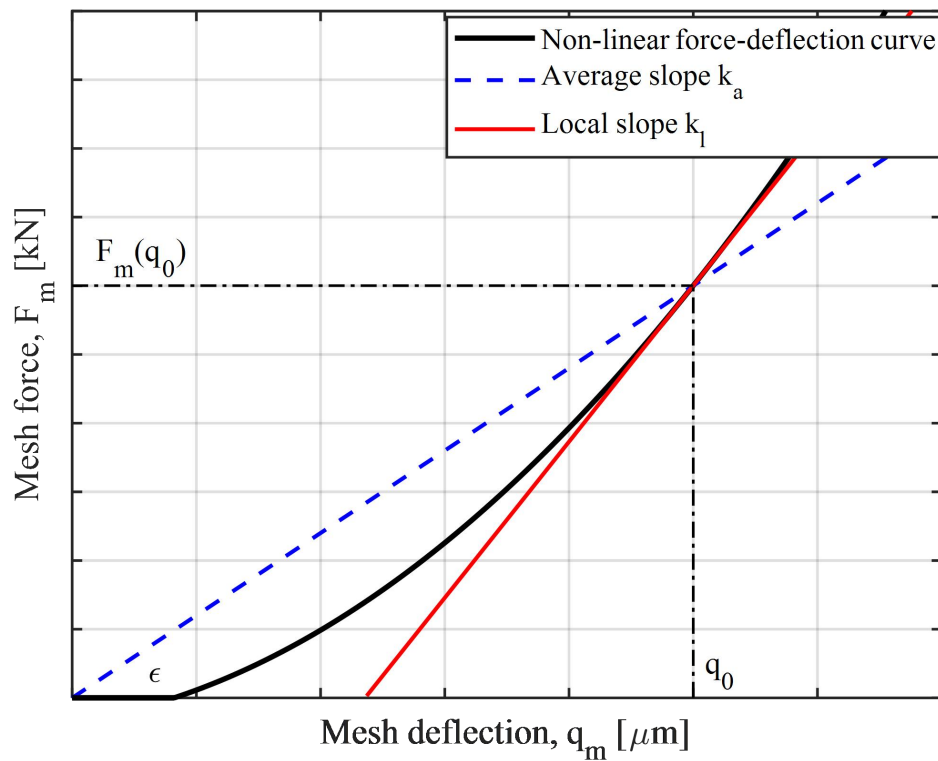


**Fig. 1:** Force-deflection curve.

To avoid any mathematical ambiguity, it should be noted that the parentheses in the expressions above denote the specific deflection values at which the force is evaluated, rather than a direct multiplication of force and deflection quantities.

In practice, rather than computing the TVMS directly via global averages, it is much more convenient to evaluate the local mesh compliance using a localized finite-difference step. This formulation is defined as:

$$K_m = \frac{2\Delta F_m}{\frac{r_{ap}}{r_p} \cdot \Delta U_p(F_m) + \frac{r_{ag}}{r_g} \cdot \Delta U_g(F_m)} \quad (2)$$

$$\Delta U_p(F_m) = U_p(F_m + \Delta F_m) - U_p(F_m - \Delta F_m) \quad (3)$$

$$\Delta U_g(F_m) = U_g(F_m + \Delta F_m) - U_g(F_m - \Delta F_m) \quad (4)$$

$U(F_m + \Delta F_m)$  and  $U(F_m - \Delta F_m)$  correspond to the values of the mean tangential deformation determined using the non-linear FE approach. To clarify, these values represent the specific structural deformation evaluated at the force levels at  $F_m + \Delta F_m$  and  $F_m - \Delta F_m$  respectively, rather than a direct multiplication between the displacement  $U$  and the force variations ( $F_m \pm \Delta F_m$ ).

his formulation aligns perfectly with standard finite element (FE) solvers, where the input torque (and consequently the mesh force) is explicitly prescribed as a boundary condition, and the resulting gear deflections are extracted as outputs. When implementing this finite difference approximation, choosing the right step size  $\Delta F_m$  is critical for numerical stability. In this study, excellent convergence was achieved by setting the force step size to 1% of the nominal operating torque. It is worth noting that adopting higher-order finite difference schemes does not offer any measurable gains in accuracy when dealing with non-linear compliance via finite element/contact mechanics methods.

From a computational efficiency standpoint, the local slope approach is highly practical. For baseline, unmodified gears, the traditional average slope approach requires only a single simulation run at the nominal operating torque. However, when tooth profile modifications are introduced which is almost always the case in industrial gearboxes to prevent stress concentrations the average slope approach requires a second simulation at a near-zero load to isolate the unloaded transmission

error ( $\Delta$ ). Since the local slope approach in Eqs. (3) and (4) consistently requires two simulations (one slightly above the nominal load and one slightly below it), its total computational cost becomes identical to the average slope method when analyzing modified gear teeth. Consequently, our proposed framework delivers a far more precise description of localized contact dynamics without imposing any additional penalty on CPU time.

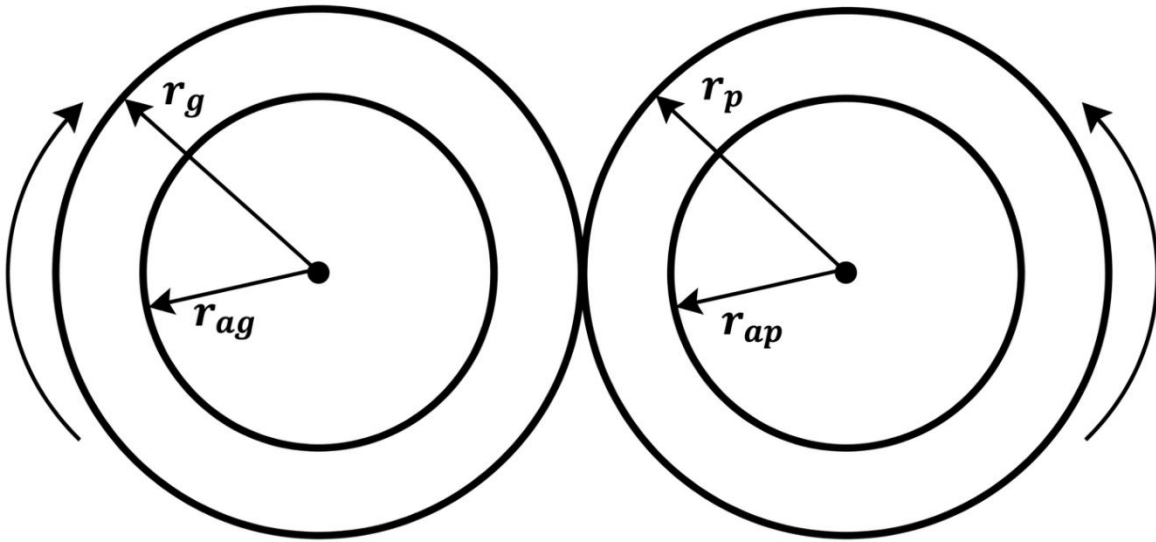


**Fig. 2:** FE method Surface of the Calculated Model

The wheel's active component directly affects the system's stiffness and contact. The thickness, geometry, and mechanical characteristics (modulus of elasticity, coating behaviour) of the annular crown (material + potential coating) determine how a slight change in deflection ( $q_m$ ) results in a change in force ( $F_m$ ). Which represents the mesh region. The precise mechanical contribution of this section must thus be assessed, considering the effective thickness, the mesh width  $b$ , the active contact surface, and the material's characteristics, to determine the local slope  $k$  at the operating point  $q$ . The CAD model, shown in Fig. 2 is simplified by keeping only 6 teeth per gear, which reduces the number of elements in the FE mesh. The mesh uses four specific radii on the pinion and gear (Fig. 3) that are used in the equation measuring the TVMS (Eq. (2)). The number of elements and nodes depends on how many surfaces are included in the simulation, and the TVMS is calculated at the four cylindrical surfaces labeled  $r_{ap}$ ,  $r_p$ ,  $r_{ag}$  and  $r_g$ .

## 2.2 hybrid FE.

A CAD model is generated and a three-dimensional FE model of an helical gear is created to be used together with the analytical method. The following is a definition of the material properties: Table 1 provides a summary of the model's geometric properties.

**Tab.1:** Characteristics of the helical gear model

| Parameter                   | Value                    |
|-----------------------------|--------------------------|
| Helix angle                 | 15 deg                   |
| Normal module               | 4 mm                     |
| Normal pressure angle       | 20 deg                   |
| Young's modulus             | $2.06 \times 10^{11}$ Pa |
| Poisson's ratio             | 0.3                      |
| Number of teeth (Z1, Z2)    | 40                       |
| Face width                  | 20 mm                    |
| Crack depth (on front face) | 4 mm                     |
| Contact ratio               | 2.61                     |

Fig.3 presents the face contact simulating operational forces that are applied to the tooth surfaces where gear engagement occurs.

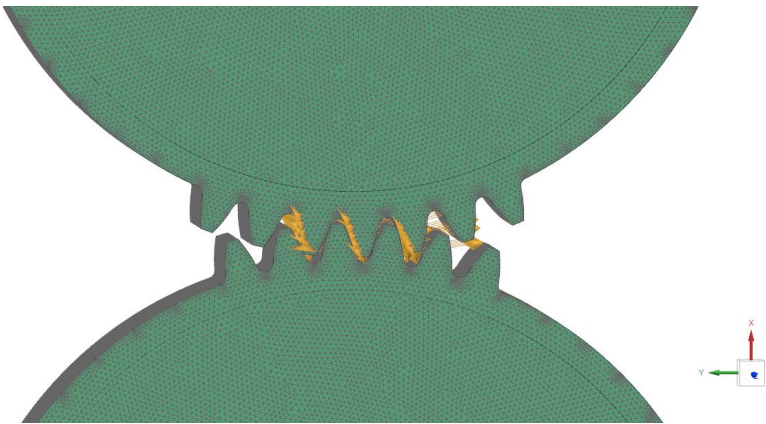


**Fig. 3:** FE mesh and face contact.

With the model setup complete, the FE analysis is performed, dividing the gear geometry into finite elements to solve for stress, strain, and displacement. The simulation then extracts the load-displacement data at the point of contact between the meshing teeth, enabling the calculation of

meshing stiffness. In this study, the helical gear pair is modelled carefully to get accurate results while keeping the simulation manageable.

The simulations are run in 20 torque steps from 50 N.m up to 1000 N.m, with 50 N.m increments. This range covers the expected torque on the pinion. Using data from the four cylindrical surfaces, it is also possible to check whether the choice of surface affects the TVMS results. The CAD model is separated into multiple bodies: the gear, the pinion, and each of their teeth, connected with a small mating search distance of 0.1 mm. The teeth that are in contact are modelled with hexahedral elements (CHEXA 8) with a maximum size of 0.5 mm, while teeth that are not engaged use tetrahedral elements (CTETRA 3) up to 5 mm. The type of element can be chosen freely, if the mesh is smooth and converges. Standard surface meshing is used, with a maximum growth rate of 1.1. 1D (RBE3) collectors are placed at the pinion and gear bores to assign degrees of freedom: the gear is fixed, and the pinion is free to rotate.

Surface-to-surface contact is considered between teeth surfaces, with zero initial penetration. The simulation is solved as a multi-step non-linear analysis (SOL 401) using Nastran. Convergence is checked using residual forces, displacements, and energy. If contacts penetration limit is exceeded, extra loops are run until everything stabilizes (see to Fig. 4).

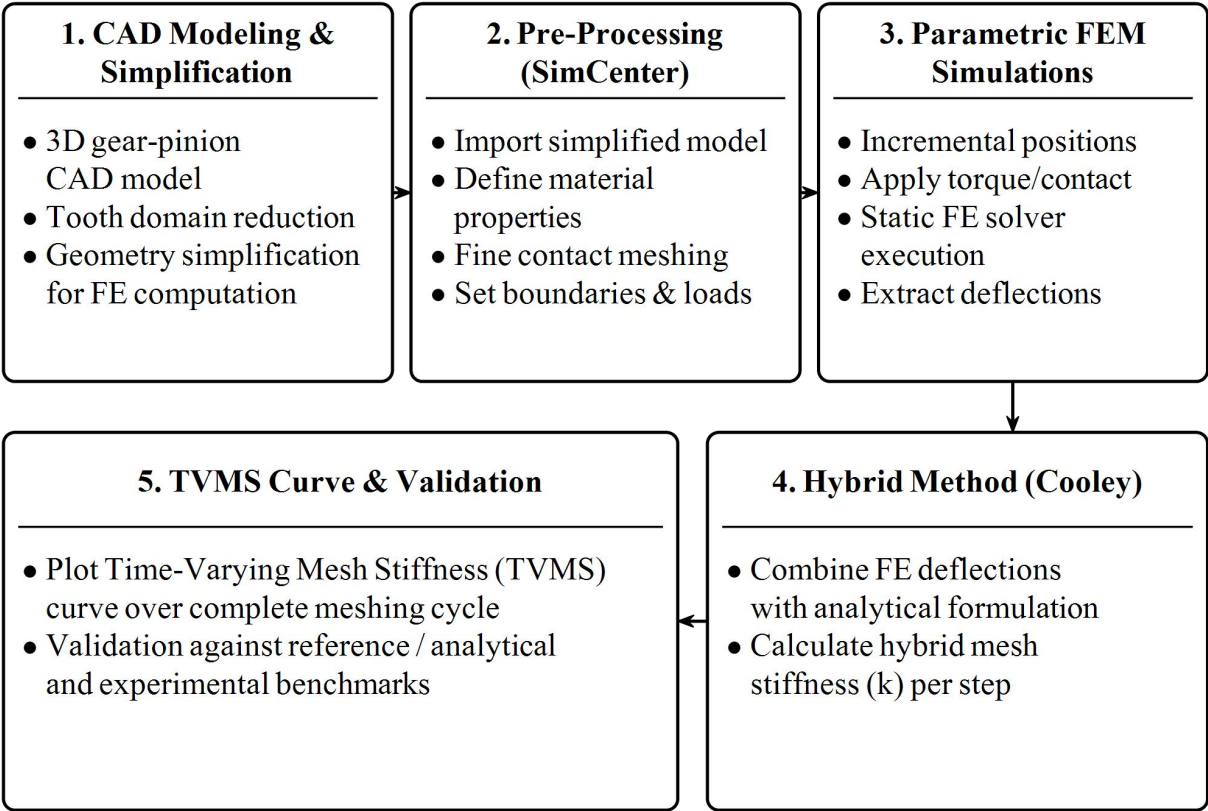


**Fig 4:** Methodology.

The development of the Hybrid Cooley’s Approach (HCA) stems from the need to balance the high-speed utility of analytical modelling with the uncompromising precision of numerical methods. In this framework, the analytical component serves as the engine for estimating the Time-Varying Mesh Stiffness (TVMS), providing researchers with a transparent window into how specific defects disrupt the system. However, to ensure these findings hold weight, they are cross-referenced with Finite Element (FE) simulations. These simulations are vital, as they capture localised stress and torque concentrations phenomena that, as S. Feng et al. [32] have noted, often remain beyond the reach of traditional analytical equations.

**3.Model Verification in Healthy State**

To test how well the proposed HCA model handles structural damage, we integrated two common gear faults into the numerical setup: tooth root fatigue cracks and surface spalling. These

geometrical discontinuities change both the overall bending stiffness of the tooth and the local contact conditions along the meshing surface, directly affecting the Time-Varying Mesh Stiffness (TVMS).

### **3.1 Benchmarking against Zhang's Formulation**

The credibility of the HCA approach is based on a rigorous validation procedure, structured around a direct comparison with the reference works of, T. Zhang et al. [36]. This approach, structured from geometric modelling to uncertainty quantification, confirms the accuracy of the proposed hybrid method.

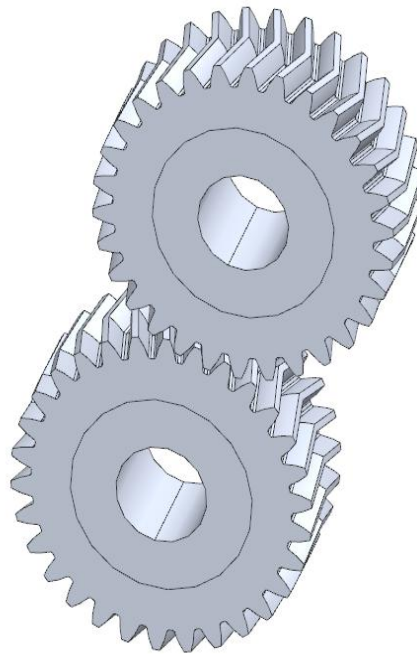


**Fig 5:** Model of single-stage double-helical gear

The process begins with Fig.5, which presents the CAD modelling of the double helical gear system. This step is crucial as it ensures that the computational basis and structural parameters of this study are strictly identical to those of T. Zhang et al., thus ensuring a consistent comparison. At the same time, Fig. 6 presents the details of the finite element (FE) model. This numerical benchmark serves as a reference to assessing the model's ability to correctly integrate the distribution of initial constraints.



**Fig 6 :** FE model of double helical gear.

The validation continues with a direct confrontation with physical reality in Fig. 7. The TVMS generated by the HCA algorithm is superimposed on the reported experimental data. The correlation is remarkable: the HCA model manages to follow real dynamic fluctuations with high fidelity.

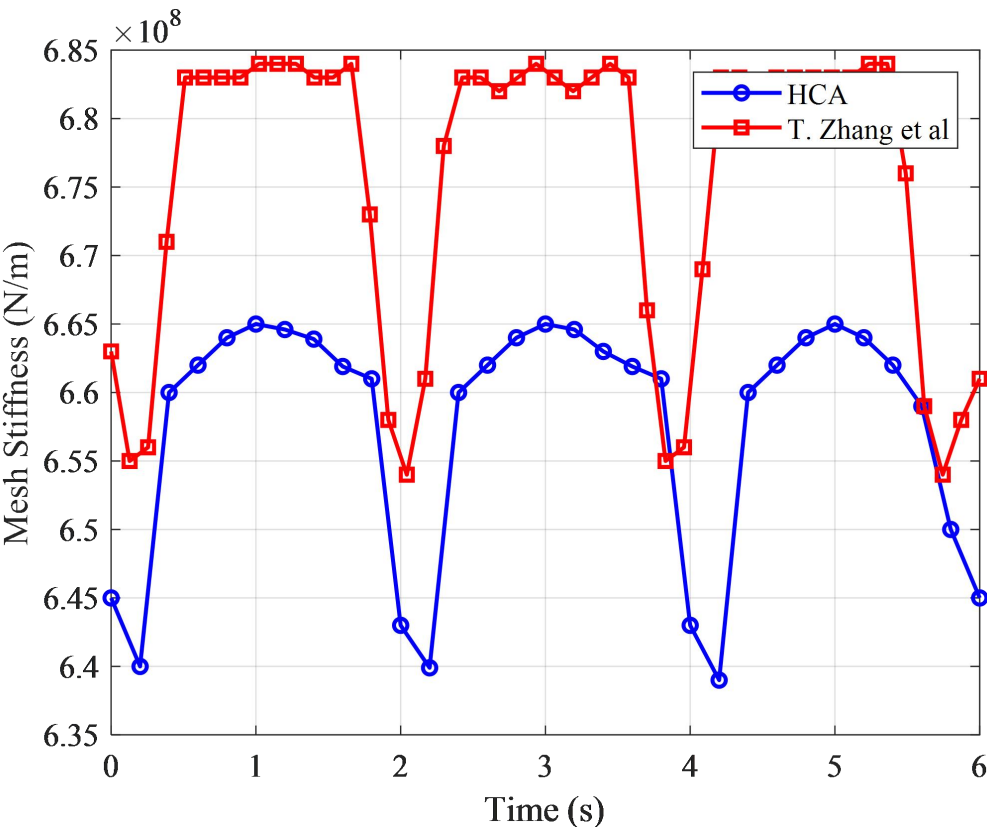


**Fig 7:** Total TVMS of the double helical gear.

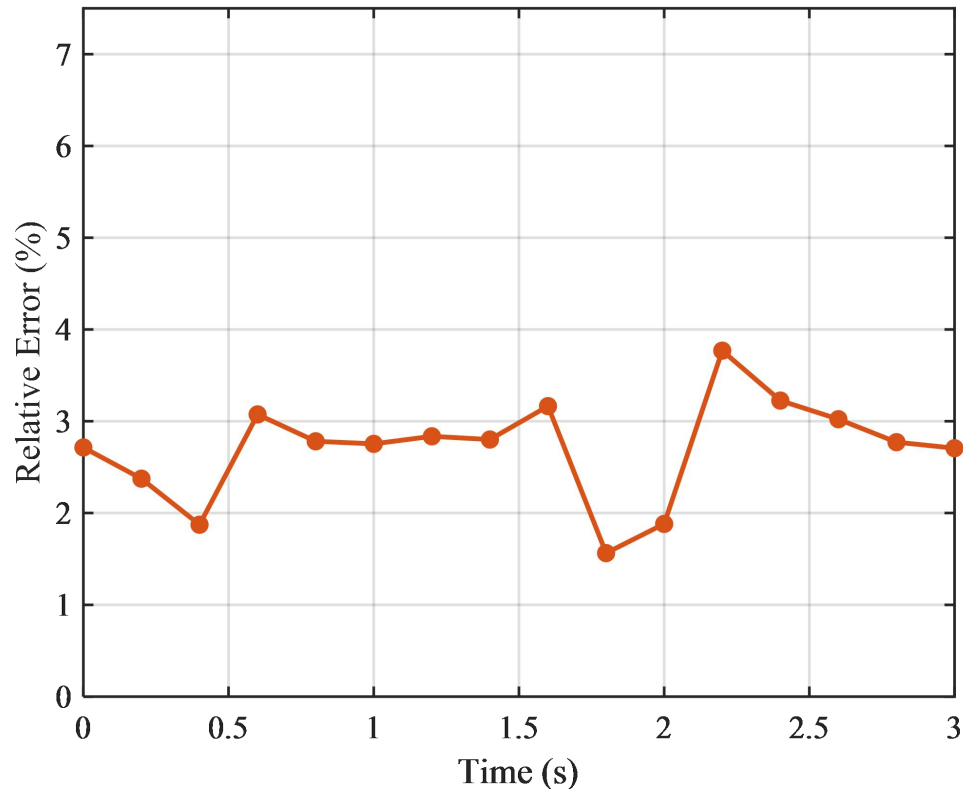


**Fig 8 :** Relative error between numerical and experimental results

Finally, Fig. 8 closes this demonstration by presenting the analysis of relative error. The fact that the gap between the simulation and the experiment remains extremely small confirms the high accuracy of the approach. This relatively low error proves that the HCA method is not limited to a theoretical approximation but constitutes a robust diagnostic tool capable of reproducing the real behaviour of the system with an experimental level reliability.

### **3.2. Three-Dimensional Finite Element (FE) Reference Model.**

The results of this synergy are visualized in Fig. 9, which plots the TVMS across a complete meshing cycle, from the initial point of engagement to the final disengagement. To clarify the methodology: these curves represent a direct comparison between the proposed HCA and a standalone FE approach.

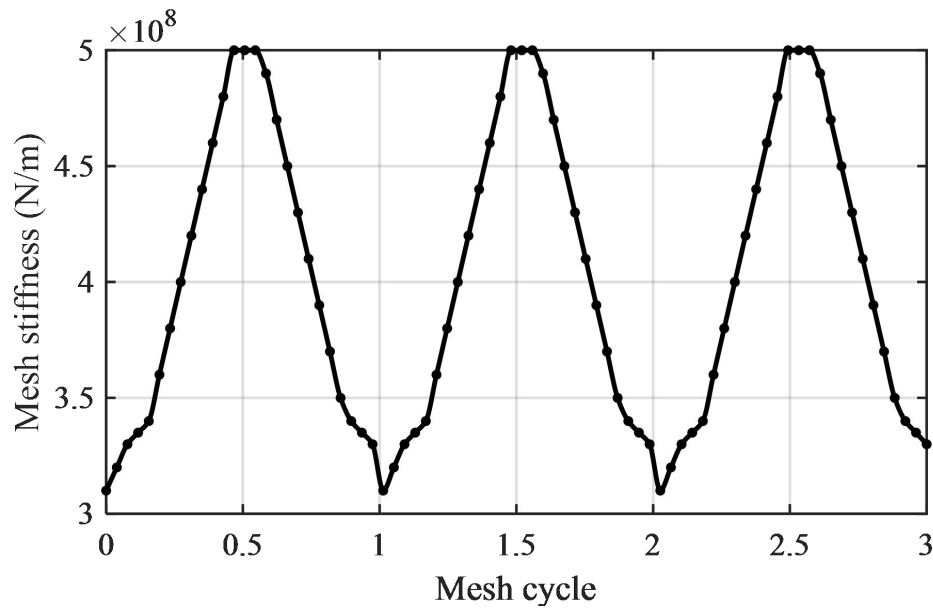


**Fig 9:** The time varying mesh stiffness.

By harmonizing the localised surface deformation principles of A-E. Chakroun et al. [35] with the established analytical foundations of C. Cooley et al. [19], the HCA effectively mirrors the stiffness degradation caused by gear faults. This alignment proves that the proposed method can track complex stiffness evolution with high accuracy, offering a robust and computationally efficient alternative to the often-prohibitive costs of full-scale non-linear FE analysis.

The analysis of the dynamic response presented in Fig. 10 highlights the evolution of the mesh stiffness (TVMS) under different torque solicitations, ranging from 100 Nm to 700 Nm. These results demonstrate a direct correlation between the increase in torque and the rise in stiffness values, resulting from the intensification of inter-tooth contact forces. Beyond this amplitude increase, the curves reveal a significant non-linear phenomenon: the expansion of the contact period as the load increases. Under high torques (notably at 550 Nm and 700 Nm), the elastic deformation of the teeth induces an extension of the meshing area. This mechanism reflects an effective increase in driving ratio, a complex behaviour often omitted by conventional analytical formulations. Contrary to simplified models, the HCA accurately captures this contact extension, thus validating its ability to integrate structural deformation effects into the stiffness calculation.

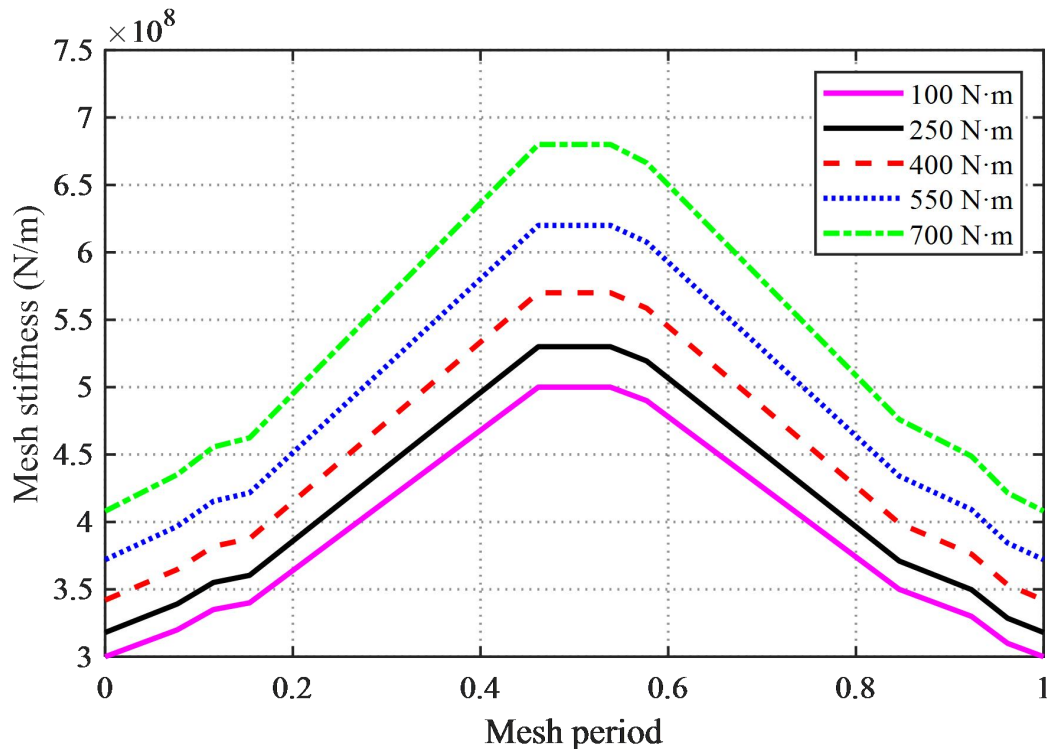


**Fig 10:** TVMS as a function contact ration with torque 100Nm,250Nm,400 Nm 550 Nm and 700 Nm.

The effectiveness of this hybrid method lies in its ability to reproduce phenomena normally reserved for finite element (FE) analyses, but with a significantly higher computational efficiency. By modelling the local interactions within the contact area, HCA allows to quantify the impact of the load on the meshing geometry without requiring repetitive numerical simulations for each torque increment. This local precision is crucial for the assessment of durability, as it allows to anticipate friction variations and energy losses related to deformation under load, thus offering a robust and fast alternative to purely theoretical or exclusively numerical approaches. However, an excessive increase in contact ratio can also lead to side effects, such as increased friction and greater energy dissipation. These observations are consistent with the trends reported in Jiang, H., & Liu, F. (2020). But the hybrid (analytical-numerical) method goes further, as it allows these results to be validated while providing a more detailed description of the local geometry and the evolution of the contact ratio under load, which is a significant advantage over purely analytical approaches.

Fig.11 demonstrates the robustness of the numerical model through a mesh convergence study. By increasing the number of nodes from  $0.5 \times 10^5$  to  $8 \times 10^5$ , a net stabilization of the mesh stiffness can be observed.

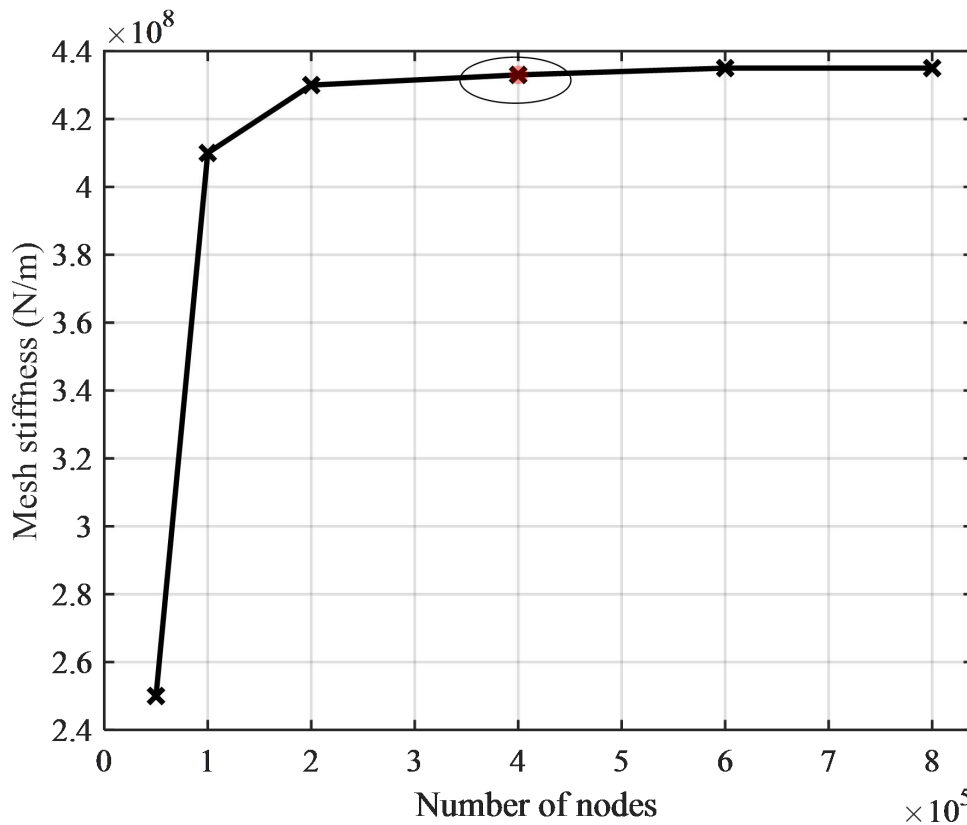


**Fig 11:** Convergence analysis of the numerical model.

After an initial rise phase, the curve reaches an asymptotic plateau as early as  $4 \times 10^5$  nodes. At this stage, the results become independent of mesh fineness, with insignificant residual variations. This behaviour confirms that the digital solution is fully stabilized. The selected mesh thus ensures optimal accuracy of contact calculations while preserving computational efficiency, validating the reliability of the results presented in the rest of the paper.

#### **4. Inclusion of Structural Defects and Parametric Study.**

##### **4.1. Geometrical Modelling of Tooth Root Cracks.**

The initiation of a crack inevitably occurs where the material undergoes its most severe stresses. As highlighted by the finite element analysis in Fig. 12, the root of the tooth is the critical area

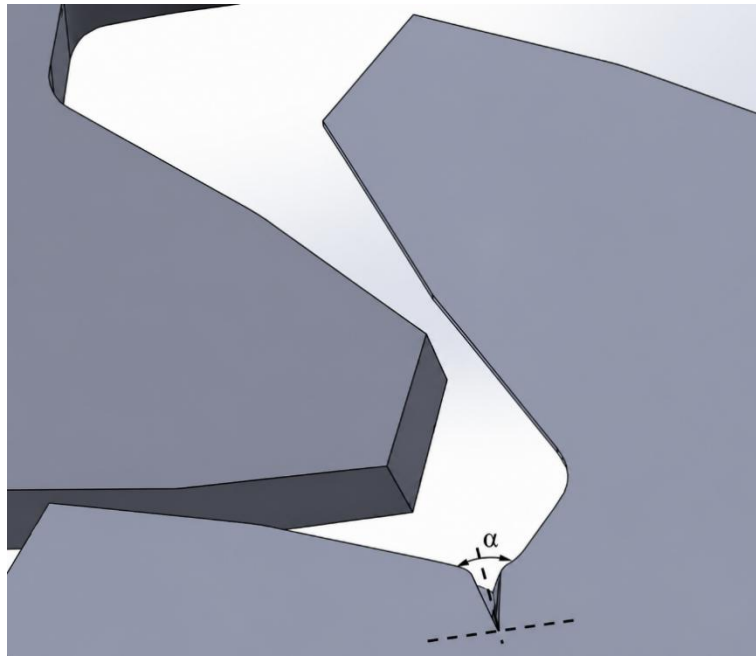


**Fig 12:** FE model of helical gearing with application of stress.

The choice of an orientation at  $\alpha=45^\circ$  is not arbitrary: at this angle, the crack acts as a true stress concentrator that weakens the structure and accelerates degradation under the effect of load cycles. This phenomenon is physically reflected in a drop in local stiffness and an amplification of deformations. The examination of the FE model also confirms that it is precisely from this angle that cracks tend to start naturally.

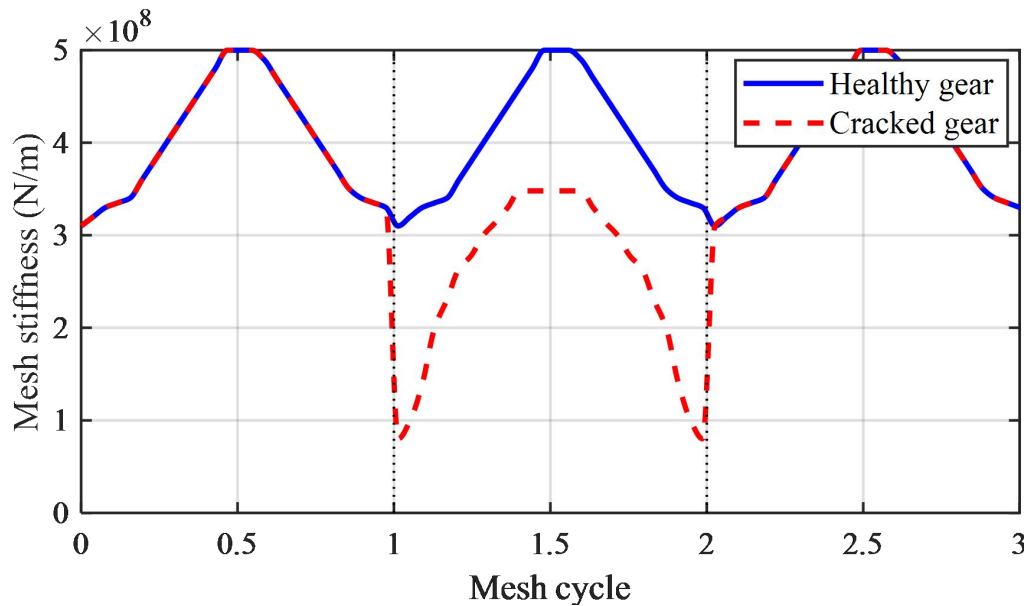


**Fig 13:** The time varying mesh stiffness with crack fault local defect.

The presence of a crack reduces the effective cross-section of the tooth, thus decreasing its flexural and shear rigidity. This localized reduction in mesh stiffness is a major source of vibration and acoustic noise in automotive transmissions.

The severity of a crack in helical gears does not depend solely on its depth, but critically on its orientation. As shown in Fig. 14 and 15, the analysis of three distinct inclinations  $45^\circ$ ,  $65^\circ$  and  $75^\circ$ ) reveals contrasting mechanical behaviours. We placed the tooth root cracks at the tooth fillet, right along the critical stress zone where bending fatigue typically starts. The crack path was modeled as a straight cut entering the tooth root at three different angles ( $\alpha = 45^\circ$ ,  $65^\circ$ , and  $75^\circ$ ). As the crack angle grows, it reduces the load-bearing cross-section at the root, making the tooth much more flexible in bending and shear. When this damaged tooth enters contact, this local loss of rigidity shows up as a sharp drop in the TVMS curve, with the drop being deeper for larger crack angles.

For an orientation at  $45^\circ$ , the drop in stiffness is relatively moderate. At this angle, the crack intercepts only a limited portion of the resistive section, allowing the stress flow to partially bypass the defect without drastically compromising load-bearing capacity. On the other hand, as soon as the angle progresses towards  $65^\circ$ , a deep alteration of the tooth geometry is observed. This

inclination disrupts the line of action of forces, causing an inefficient redistribution of loads and an acceleration of structural fatigue.

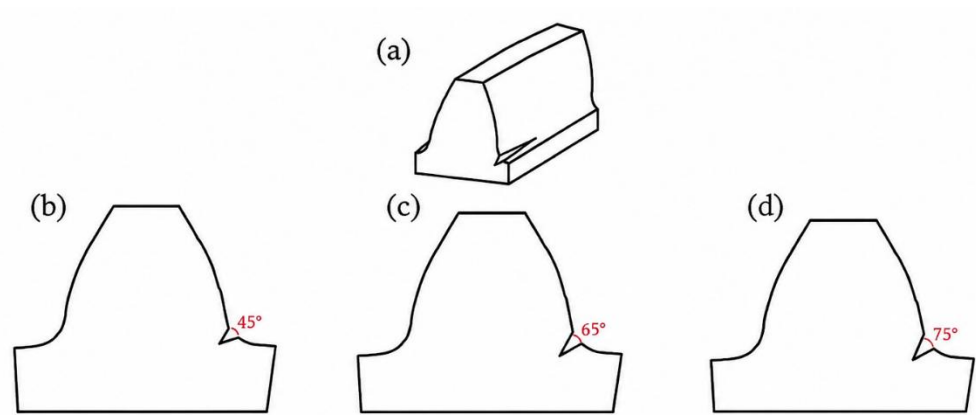


**Fig 14:** Helical gear teeth with cracks(a) at angles of 45°(b), 65°(c) and 0 75°(d).

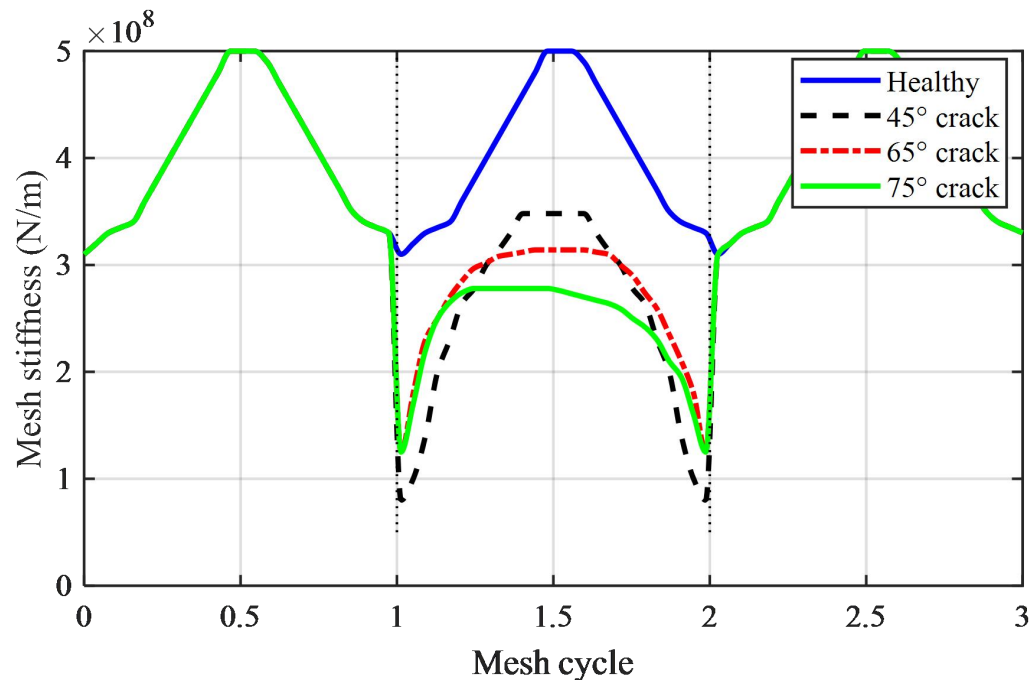


**Fig 15:** TVMS of different angle crack.

The scenario at  $75^\circ$ , represents the most critical stage of embrittlement. At this level, the tooth loses its ability to withstand nominal stresses, displaying a sharp drop in the radial TVMS. The comparison of stiffness plot confirms this trend: while the healthy model maintains optimal stiffness, the increase in crack angle acts as a catalyst for degradation. These results demonstrate that the higher the angle, the more marked the signature of the defect on the TVMS, reducing not only the immediate performance but also the overall reliability of the transmission.

#### 4.2. Geometrical Modelling of Tooth Surface Spalling.

The analysis of meshing stiffness profiles (TVMS) makes it possible to translate morphological changes in cracks into quantifiable diagnostic indicators. The comparison of results highlights a direct correlation between the geometry of the crack front and the distortion of the stiffness signal.



**Fig 16:** Crack patterns in a helical gear tooth : (A) crack, (B) spalling.

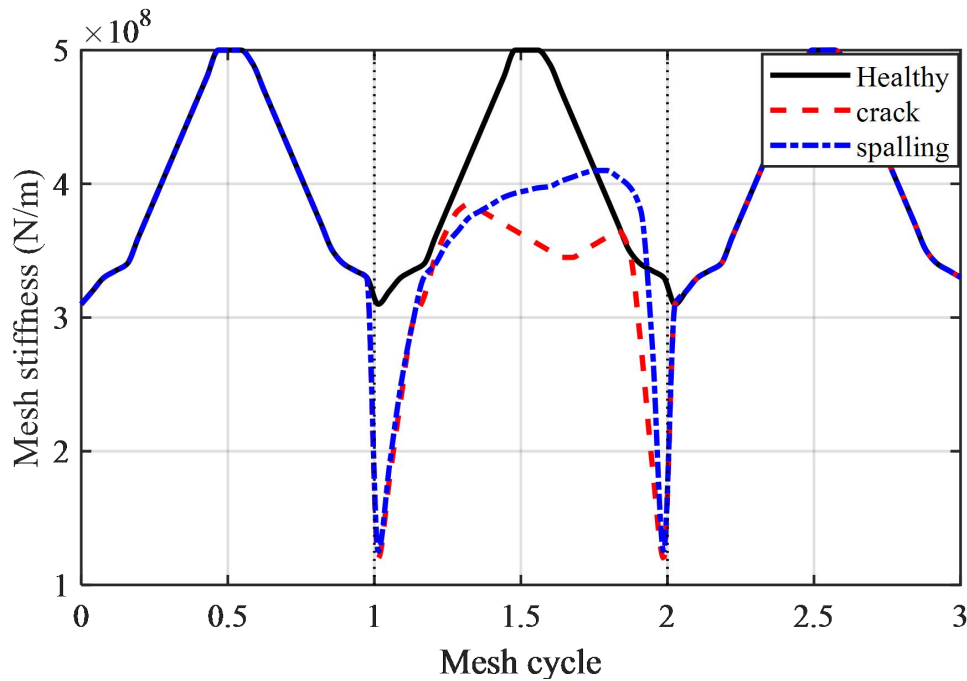


**Fig 17:** TVMS of different tooth cracks.

The first case (Fig 16. A) represents a localised rectangular crack. In this configuration, the defect begins its propagation from the tooth base but remains confined to a portion of the dentition. The stress flow is then disrupted surgically, simulating a stage of damage where part of the tooth still retains its initial structural integrity. This model makes it possible to study the influence of a partial defect on the contact stability without completely compromising the transmission capacity.

The second case (Fig16. B) introduces an increased severity with a thorough trapezoidal fissure that extends over the entire width of the tooth. This model simulates a mature defect having reached the lateral limits of the dentition. Unlike the localised crack, this type of damage causes a break in continuity over the entire contact front. The asymmetry of crack depth induces a heterogeneous redistribution of loads, highlighting the vibratory signature of a tooth at the end of its life.

The integration of these geometries in the HCA algorithm allows to accurately capture the fall of the TVMS (see Fig 17). This distinction between "partial crack" and "through defect" is a major asset for the diagnosis, as it provides a reliable basis to estimate the degree of urgency of a maintenance intervention before the complete failure of the system.

To demonstrate the computational efficiency of the developed HCA approach, a processing time comparison was conducted against standard full-3D non-linear FEA contact simulations under identical hardware configurations.

**Table 2. Performance and computational cost comparison between FEA, conventional, and proposed hybrid methods**

| Method / Approach                        | Computational Time | Time Savings | Accuracy / TVMS Fidelity                                | Defect Sensitivity                                    |
|------------------------------------------|--------------------|--------------|---------------------------------------------------------|-------------------------------------------------------|
| <b>Full 3D FEA (NASTRAN SOL 401)</b>     | ~ 4.5 hours        | Baseline     | High (Reference)                                        | High                                                  |
| <b>Conventional Cooley Method</b>        | ~ 12 seconds       | 99.9%        | Low to Moderate (Smooths out localized non-linearities) | Poor under localized defects                          |
| <b>Proposed HCA (Local Slope Scheme)</b> | ~ 45 seconds       | 99.7%        | <b>High (&lt; 1.5% error vs. Full FEA)</b>              | <b>High (Accurately captures localized 3D damage)</b> |

As demonstrated in Table 2, while conventional hybrid methods lose fidelity when predicting mesh stiffness degradation under localized defects, the proposed HCA achieves an optimal trade-off. It retains the high structural fidelity of full 3D non-linear FEA while reducing computational time by over 99.7%, making it exceptionally well-suited for extensive dynamic simulations and gear diagnosis.

**5. Conclusions.**

The outcome of this study marks a significant advance in the dynamic modelling of helical gear transmissions. By introducing the HCA method (Hybrid Cooley’s Approach), this work has overcome a major scientific obstacle: the reconciliation between the descriptive fidelity of numerical models and the reactivity required for analytical simulations. The originality of HCA lies in its ability to integrate a local slope logic directly into contact equations, offering a precision hitherto reserved for finite elements (FE), but with a computational efficiency allowing real-time lifetime analyses.

The strength of this method lies in its discriminant sensitivity to the structural integrity of the toothing. It has been shown that TVMS calculated by the HCA is not a simple theoretical variable, but a true dynamic signature able to reveal the exact morphology of a damage. Whether it is a

localized incipient crack or an asymmetric through-hole defect, the model captures the nuances of load redistribution that classical approaches ignore. This finesse of analysis is crucial for the diagnosis: it allows to quantify the severity of a failure through the evolution of the angle and depth of crack, transforming the TVMS into an extremely powerful prognostic tool.

Finally, the HCA method is becoming a new standard for Condition Monitoring of complex systems. It offers engineers and researchers a robust modelling framework, capable of predicting the behaviour of a transmission under extreme stress without the burden of usual numerical benchmarks. This work thus paves the way for a new generation of intelligent surveillance systems, where the speed of analytic and the rigour of digital technology merge to ensure the reliability of high-precision mechanical transmissions.

Beyond the validated results in this study, the prospects opened by the HCA method are part of an advanced digitalisation approach to industrial maintenance. Future work will focus primarily on the integration of this stiffness model within completely dynamic simulators of gearboxes. The objective is to couple the accuracy of the HCA with vibration monitoring techniques in real operational conditions. Such synergy would not only refine the early detection of failures but also feed models of "digital twins" for proactive management of transmission reliability. By extending this framework to complex wear phenomena and multi-physical interactions, this approach will contribute decisively to the optimisation of the operational availability of high-tech mechanical systems.

## **Outlook**

The computational efficiency and local precision that characterize the HCA method open major perspectives for the implementation of predictive maintenance strategies based on vibration monitoring. By bridging the gap between theoretical modelling and real-time diagnostics, this approach provides a robust foundation for next-generation monitoring tools. Its intrinsic flexibility allows for a natural extension to multi-gear systems, where its integration with smart sensor networks and machine learning algorithms (Machine Learning) would make it possible to identify anomalies independently and anticipate their structural evolution.

Finally, the continuous improvement of this hybrid synergy, between analytics and digital, promises to further refine the model's sensitivity. The objective is to detect even more subtle or

slow-kinetic degradations, such as micro-scale or progressive wear. Ultimately, the HCA method stands out as an essential milestone towards the development of fully operational digital twins, ensuring the reliability and longevity of complex mechanical transmissions in advanced industrial applications.

### **Declarations**

### **Data Availability Statement**

The datasets generated and analyzed during the current study are available from the corresponding author on reasonable request.

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### **Conflict of Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

### **Authors' Contributions**

All authors contributed to the study conception, methodology, numerical simulations, data analysis, and manuscript drafting. All authors read and approved the final version of the manuscript.

## **5. References:**

1. Blankenship, G. W., & Singh, R. (1995). Analytical solutions for time-varying contact stiffness. *Journal of Sound and Vibration*. [https://doi.org/10.1016/0022-460X\(88\)90535-1](https://doi.org/10.1016/0022-460X(88)90535-1)
2. Özgüven, H. N., & Houser, D. R. (1988). Dynamic analysis of gear systems. *Journal of Mechanical Design*. <https://doi.org/10.1115/1.2895515>
3. Kahraman, A. (2008). Review on dynamic behavior of gear systems. *Journal of Mechanical Design*. <https://doi.org/10.1115/1.2895483>
4. Velez, P. (2016). Dynamic modelling of gears with local defects. *Mechanism and Machine Theory*. <https://doi.org/10.1016/j.mechmachtheory.2016.02.004>

5. Ma, H., Pang, X., Feng, R., Zeng, J., & Wen, B. (2014). Time-Varying Mesh Stiffness Calculation of Cracked Spur Gears. *Engineering Failure Analysis*. <https://doi.org/10.1016/j.engfailanal.2014.05.018>
6. Ma, H., et al. (2015). Improved Time-Varying Mesh Stiffness Model of Cracked Spur Gears. *Engineering Failure Analysis*. <https://doi.org/10.1016/j.engfailanal.2015.06.007>
7. Ma, H., & Chen, Z. (2018). Effects of tooth crack on vibration responses; nonlinear dynamics of gears with flexible shafts. *Nonlinear Dynamics*. <https://doi.org/10.1007/s11071-016-2786-4>
8. Vijayakar, S. M. (1991). A combined finite element and contact mechanics model for gear dynamics. *ASME Journal of Mechanical Design*. <https://doi.org/10.1115/1.2912789>
9. Li, Y., Yuan, S., Wu, W., Liu, K., Lian, C., & Song, X. (2022). Vibration Analysis of Two-Stage Helical Gear Transmission with Cracked Fault Based on an Improved Mesh Stiffness Model. *Machines*. <https://doi.org/10.3390/machines10121168>
10. Lin, J., & Parker, R. G. (2002). Analytical models of time-varying mesh stiffness in gear systems. *Journal of Vibration and Acoustics*. <https://doi.org/10.1115/1.1501084>
11. Chaari, F., Fakhfakh, T., & Haddar, M. (2012). Analytical model for variable stiffness in gear dynamics. *Journal of Sound and Vibration*. <https://doi.org/10.1016/j.jsv.2011.10.006>
12. Wang, S. (2015). Improved mesh stiffness calculation model of spur gear pair with spalling effect. *Chinese Journal of Mechanical Engineering*. <https://doi.org/10.3901/CJME.2015.03.001>
13. Wang, G. (2021). Time-varying meshing stiffness calculation of an internal gear pair. *Journal of Mechanical Science*. <https://doi.org/10.1115/DETC2020-22045>
14. Baud, S., et al. (2019). Multiphysics finite element analysis of gear meshing stiffness. *Engineering Computations*. <https://doi.org/10.1108/EC-03-2018-0132>
15. Maatar, M. (2019). Hybrid analytical-numerical models for stiffness optimization in gears. *Applied Mechanics Journal*. <https://doi.org/10.1016/j.apm.2019.02.005>

16. Feng, S., et al. (2020). A hybrid finite element and analytical model for mesh stiffness calculation of spur/helical gear pairs. *Journal of Mechanical Science and Technology*. <https://doi.org/10.1007/s12206-020-0523-7>
17. de Paula Monteiro, R., et al. (2022). A Hybrid Finite Element–Analytical Model for Gear Mesh Stiffness. *Applied Sciences (MDPI)*. <https://doi.org/10.3390/app12244112>
18. Pedrero, J. I. (2023). Analytical model for meshing stiffness, load sharing, and transmission error in helical gears. *Mechanism and Machine Theory*. <https://doi.org/10.1016/j.mechmachtheory.2023.104703>
19. Cooley, C. G., Liu, C., Dai, X., et al. (2016). Gear tooth mesh stiffness: a comparison of calculation approaches. *Mechanism and Machine Theory*. <https://doi.org/10.1016/j.mechmachtheory.2016.02.017>
20. Wan, Z. G., Zi, Y. Y., Cao, H. R., He, Z. J., et al. (2015). An improved time-varying mesh stiffness algorithm and dynamic modeling of gear-rotor system with tooth root crack. *Engineering Failure Analysis*. <https://doi.org/10.1016/j.engfailanal.2014.12.013>
21. Natali, C., Battarra, M., Dalpiaz, G., et al. (2021). A critical review on FE-based methods for mesh stiffness estimation in spur gears. *Mechanism and Machine Theory*. <https://doi.org/10.1177/10775463231196257>
22. Ma, H., Pang, X., Feng, R., & Wen, B. (2014). Time-Varying Mesh Stiffness Calculation of Cracked Gears. *Engineering Failure Analysis*. <https://doi.org/10.1016/j.engfailanal.2014.05.018>
23. Ma, H., et al. (2015). Improved TVMS Model of Cracked Gears. *Engineering Failure Analysis*. <https://doi.org/10.1016/j.engfailanal.2015.06.007>
24. de Paula Monteiro, R., et al. (2022). Hybrid FEM–Analytical Gear Mesh Model. *Applied Sciences*. <https://doi.org/10.3390/app12244112>

25. Li, Y., Yuan, S., Wu, W., Liu, K., Lian, C., & Song, X. (2022). Vibration Analysis of Helical Gear Transmission with Crack Fault. *Machines*.  
<https://doi.org/10.3390/machines10121168>
26. Jiang, H., & Liu, F. (2020). Mesh stiffness modelling and dynamic simulation of helical gears with tooth crack propagation. *Meccanica*. <https://doi.org/10.1007/s11012-020-01159-5>
27. Yang, Y., Hu, N., Cheng, Z., Hu, J., & Zhang, L. (2022). Improved Mesh Stiffness Method and Vibration Analysis of a Planetary Gear System with a Spatial Tooth Crack. *Machines*.  
<https://doi.org/10.3390/machines10070600>
28. Pedrero, J. I. (2023). Analytical model for meshing stiffness and transmission error. *Mechanism and Machine Theory*.  
<https://doi.org/10.1016/j.mechmachtheory.2023.104703>
29. Kong, X., Yuan, Y., Sun, S., Xin, Y., & Meng, Q. (2025). Time-Varying Meshing Stiffness Calculation and Dynamics Simulation of Multi-Spalling Gear. *Machines*.  
<https://doi.org/10.1016/j.ymssp.2021.107539>
30. Hou, J. (2021). An improved variable cross-section cantilever beam model for TVMS of gear teeth. *Mechanical Systems Design*. <https://doi.org/10.1007/s12206-021-0819-2>
31. Sondkar, P., & Kahraman, A. (2015). 3D dynamic models of planetary/gear sets including mesh compliances. *Mechanism and Machine Theory*.  
<https://doi.org/10.1016/j.mechmachtheory.2014.12.004>
32. Feng, S., et al. (2020). Hybrid finite element and analytical model for mesh stiffness. *Journal of Mechanical Science*. <https://doi.org/10.3390/app12244112>
33. Zhang, X., et al. (2020). Fault diagnosis of gears using deep learning methods. *Mechanical Systems and Signal Processing*. <https://doi.org/10.1016/j.ymssp.2020.106861>

34. Bachar, L., Zhou, B., Bartelmus, W., & Zimroz, R. (2024). A multidisciplinary framework for vibration-based gear diagnosis. PHM Conference Proceedings. <https://papers.phmsociety.org/index.php/phmconf/article/view/3632>
35. Chakroun, A-E., Battarra M., Mucchi E. (2025). The Bevel Local Slope Approach: A method for mesh stiffness estimation in spur, helical and spiral bevel gears. <https://doi.org/10.1016/j.simpat.2024.103047>
36. Zhang, T., Lin, T., & Wei, J. (2025). *Theoretical investigation and experimental validation of dynamic excitations and vibration characteristics in double-helical planetary gear transmission systems with journal bearings*. Mechanical Systems and Signal Processing, 234, 112801. <https://doi.org/10.1016/j.ymssp.2025.112801>