

Sensitivities of Transverse and Torsional Vibrations to Gear Tooth Crack Growth: A Numerical and Experimental Comparison

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Abstract

Vibration measurement and signal analysis are widely employed techniques for gearbox fault diagnosis in industry, typically including transverse and torsional vibration measurements. A key advantage of torsional vibration compared to transverse vibrations is its simpler frequency spectrum. This is because it is not affected by amplitude modulation caused by the time-varying vibration transmission path. This study investigates the relative sensitivities of transverse and torsional vibrations to progressive local tooth-root crack growth in a fixed-shaft spur gearbox. In addition to the spectral advantages previously associated with torsional measurements, the study examines whether torsional vibration provides a stronger response to crack progression than transverse vibration under a fixed-shaft configuration. To this end, the transverse and torsional vibrations of a gearbox drivetrain are numerically simulated using a lumped-parameter model. The transverse response is represented by pinion acceleration, while the torsional response is characterized by the dynamic torque transmitted through the pinion shaft. The time-varying mesh stiffness of the gears is calculated using the potential energy method for two cases of healthy and faulty gearboxes with a local crack of varying lengths in a tooth. The crack is made at the tooth root, and the system's dynamic response is evaluated as the crack grows up to complete tooth breakage. Various time-domain indicators have been utilized to compare the sensitivity and performance of torsional and transverse vibrations in crack detection. The results reveal higher sensitivity of torsional vibrations to fault progression. The changes in the indicators with crack growth are greater for torsional vibrations than for transverse vibrations. Experimental results further support this increasing trend for torsional vibrations over transverse ones, particularly for cracks exceeding 50%, indicating the advantage of torsional vibrations in detecting faults in fixed-shaft gearboxes.

Keywords: Torsional and transverse vibrations measurement, Sensitivity, Fault diagnosis, Early crack detection, Gear

1. Introduction

Unexpected failure of industrial machinery results in unplanned downtime in production and leads to financial losses. To prevent such incidents and enhance machinery efficiency, predictive maintenance strategies are adopted. The basis of such strategies is condition monitoring based on oil analysis, temperature monitoring, and vibration analysis. Among these, vibration analysis is the most commonly used condition monitoring method due to its wide applicability in machinery fault diagnosis [1]. Gearboxes are critical components in industrial systems, and their proper operation is vital, motivating increased attention toward their condition monitoring. Gears, as the main power transmission elements within gearboxes, are particularly prone to damage. In most studies, gearbox fault diagnosis is based on transverse vibrations, typically measured by sensors mounted on the stationary parts of the machine. Another method involves measuring torsional vibrations, usually via sensors mounted on the input or output shafts [2]. The torsional dynamic response of a drivetrain can be characterized

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using different response quantities, including angular displacement, angular velocity, angular acceleration, and dynamic torque. These quantities are physically distinct but dynamically related through the torsional stiffness, damping, and inertia of the drivetrain. Previous gearbox fault-diagnosis studies have therefore employed different torsional response variables depending on the modelling and measurement approach, including angular velocity and experimentally measured shaft torque. Part of the ISO 13373-1 [3] standard for condition monitoring addresses torsional vibration measurement methods. Due to the simplicity of the frequency spectrum produced by torsional vibrations (especially in planetary gearboxes), this method has attracted attention for diagnosing faults in certain equipment. In planetary gearboxes, because of the rotation of the planet carrier, the gear mesh position varies with time. Consequently, the vibration transmission path from the mesh point to the fixed sensor location on the gearbox housing varies with time, resulting in amplitude modulation in the measured transverse vibrations. In contrast, torsional vibrations measured from gearbox shafts are unaffected by this time-varying transmission path and thus avoid introducing such additional amplitude modulation [2]. Based on this, Feng and Zhou [2] developed a model for the torsional vibration signal of planetary gearboxes that is influenced only by amplitude and phase modulation caused by gear faults. Using amplitude and frequency demodulation analysis, they successfully identified various gear faults. Their signal model assumed steady-state operating conditions. Chen and Feng [4] improved this model for varying operational conditions, focusing on sidebands around resonance frequencies (that are constant in the time-frequency domain) rather than time-varying gear mesh frequencies. They showed that fault detection is feasible using this approach. In their experiments, they used a dynamic torque transducer to measure torsional vibrations. Xue and Howard [5] derived dynamic equations for a system comprising a planetary gearbox, motor, and load with 20 degrees of freedom. They obtained the instantaneous angular velocity of the planet carrier, unaffected by transmission path modulation. Applying various signal processing techniques, they evaluated the torsional vibration signal's effectiveness in fault detection and were also able to identify fault locations. Rotary encoder signals are also unaffected by time-varying transmission path modulation and reflect torsional vibrations. Instantaneous angular velocity features extracted from encoder signals can be used for fault diagnosis. Based on the advantages of Instantaneous Angular Speed (IAS) signals, Li et al. [6] proposed a feature extraction method combining Empirical Mode Decomposition (EMD) and Autocorrelation Local Cepstrum (ALC) for diagnosing faults in multistage gearboxes. Dogan and Karpat [7] diagnosed faults in a single-stage gearbox with asymmetric teeth using dynamic transmission error. Under time-varying speed conditions, the occurrence instant of each encoder pulse varies with time, but its relative occurrence of the angular position remains constant. Based on this, Feng et al. [8] proposed an angle-domain analysis of the order spectrum based on repetitive temporal-interval reciprocals, which is not only free from the algorithmic complexity of IAS or Instantaneous Angular Acceleration (IAA) estimation but also enables fault detection in planetary gearboxes under time-varying speed conditions. Feng et al. [9] suggested estimating torsional vibrations through the spatial difference between two encoder signals and their temporal derivatives. They analyzed the torsional angular displacement, speed and acceleration estimated based on double encoders using order-spectrum analysis for fault diagnosis in planetary gearbox bearings under time-varying speed. Unlike planetary gearboxes, fixed-shaft spur gearboxes do not involve a moving gear-meshing location relative to a stationary sensor. Therefore, the additional amplitude modulation caused by a time-varying vibration transmission path, which motivates many torsional-vibration studies of planetary gearboxes, is absent. Nevertheless, crack-induced variations in the gear-mesh excitation are transmitted to the two measurement responses through different dynamic mechanisms: the transverse vibration measured on the gearbox housing is affected by the structural transmission characteristics between the gear mesh and the sensor location, whereas the torsional vibration measured on the shaft is more directly coupled to the rotational dynamics of the drivetrain. However, it remains unclear whether this difference in transmission mechanism leads to a systematically higher sensitivity of torsional vibration to progressive tooth crack growth. To the best of the authors' knowledge, this issue has not been systematically investigated through a direct numerical and experimental comparison for fixed-shaft spur gearboxes. Therefore, the present study evaluates and compares the sensitivities of transverse and torsional vibration measurements to different stages of localized tooth-root crack growth. Using dynamic equations enables assessment of how system parameters and fault growth affect output vibration responses. Parey et al. [10] developed a 6 DOF dynamic model of a gearbox with a pitted tooth, considering time-varying mesh stiffness and damping, backlash, tooth profile modification, and gear errors. Wu et al. [11] modeled a 6 DOF system with a cracked tooth, ignoring backlash, friction, manufacturing and transmission errors to focus on crack growth's effect on simulated vibration responses. Mohammed et al. [12]

developed a 12- DOF dynamic model for a single-stage gearbox, incorporating friction and gyroscopic effects and considering different crack sizes. They also compared this model with reduced-order formulations and showed that, for symmetric gear configurations, lower-order models can provide responses close to those obtained from the higher-order formulation, whereas additional degrees of freedom become more important when asymmetric gear dynamics and gyroscopic coupling are considered. These findings indicate that the required model fidelity depends on the physical effects and response characteristics under investigation. Zhao et al. [13] constructed a nonlinear 16 DOF model for a single-stage gearbox including eccentricity, friction, transmission errors and backlash. They investigated the vibration response under the influence of each factor. A key aspect of dynamic modeling is calculating the gear mesh stiffness. A common method is the potential energy approach, where a gear tooth is modeled as a cantilever beam with non-uniform cross- section. Using beam theory, the mesh stiffness is calculated [14]. Yang and Lin [15] initially considered total potential energy in a meshing gear pair as the sum of Hertzian contact, bending and compressive energies and derived the equivalent stiffness to compute elastic forces between gears. Later, Tian [16] added shear energy effects. Sainiste et al. [17] incorporated the fillet foundation energy to account for base deflection. Hence, the total potential energy includes Hertzian contact, bending, compression, shear and fillet foundation energies and the overall mesh stiffness results from these components. Another method for calculating the mesh stiffness is using the finite element method (FEM), which is a more precise approach. However, the results depend on contact tolerance, mesh density and the selected element type [14]. The presence of faults reduces mesh stiffness and this reduction depends on fault characteristics, location and severity [14]. For instance, Liang et al. [18] derived analytical expressions for mesh stiffness in both healthy and cracked teeth, considering crack initiation from the root circle and its growth with various lengths. Ma et al. [19] established a finite element model of a geared rotor system to incorporate the effects of both extended tooth contact due to gear flexibility and tooth root crack propagation on mesh stiffness characteristics. Hu et al. [20] developed a 21 DOF nonlinear dynamic model for a planetary gear set, incorporating multiple factors such as tooth crack, bearing stiffness, transmission error, and gear backlash. Using this model, they investigated the influence of these nonlinear factors on the system's inherent characteristics and fault responses to enhance the understanding of failure mechanisms and aid in the development of fault diagnosis methods. Cao et al. [21] proposed a time-varying mesh stiffness (TVMS) calculation method that accounts for the combined effects of crack, wear, and lubricating oil film stiffness. Additionally, they introduced an 8-DOF dynamic model of spur gear system incorporating evolving TVMS and time- varying side clearance excitation caused by wear. Hu et al. [22] developed a bending-torsion-axial coupled damage dynamic model for the helicopter main reducer, in which nonlinear effects, time- varying mesh stiffness, and tooth root cracks were considered. The model also incorporated the coupling among the bevel gear set, planetary gear set, and connecting shaft, and it was demonstrated that this coupling significantly alters the natural frequencies and leads to the emergence of new vibration modes. Ma et al. [23] conducted fatigue crack propagation experiments to determine the actual crack growth paths in spur gears at different damage stages and developed a dynamic model of the gear transmission system based on these experimentally identified paths. The proposed model was validated against finite element and experimental results, and two novel vibration-based indicators were introduced to accurately identify and distinguish among different stages of crack propagation. Ma et al. [24] developed a 3 DOF nonlinear dynamic model to investigate the coupling between a tooth root crack and a pitch circle crack under complex time- varying contact conditions during meshing. By incorporating four distinct engagement states and establishing a high-fidelity time-varying meshing stiffness (TVMS) database through finite element modeling of single and coupled cracks, they systematically revealed the multi-state failure mechanisms and identified reliable vibration-based indicators for the early detection of coupled faults in gear systems.

To address the above research question, the present study follows a numerical and experimental approach to compare the sensitivities of transverse and torsional vibration measurements to progressive tooth-root crack growth in a single-stage fixed-shaft spur gearbox. First, a previously established lumped-parameter dynamic model incorporating crack-dependent time-varying mesh stiffness was employed to simulate the evolution of both transverse and torsional vibration responses as the crack propagates from an intact tooth to complete tooth failure. The responses obtained from the numerical model were then evaluated using commonly used time-domain indicators, including RMS, kurtosis, and crest factor, and their relative sensitivities to different crack-growth stages are compared. Subsequently, the principal trends identified in the numerical analysis are examined

experimentally using a fixed-shaft spur gearbox. The objective of this study is to provide a direct numerical and experimental comparison of the sensitivities of transverse and torsional vibration measurements to progressive tooth-root crack growth under the considered fixed-shaft spur gearbox configuration.

The main contributions of this study can be summarized as follows:

1. (1) A direct comparison is conducted between the sensitivities of transverse and torsional vibration responses to progressive tooth-root crack growth in a single-stage fixed-shaft spur gearbox.
2. (2) A crack-dependent time-varying mesh stiffness model is incorporated into a lumped-parameter dynamic model to investigate the evolution of both transverse and torsional vibration responses over multiple crack-growth stages.
3. (3) The relative performance of RMS, kurtosis, and crest factor is evaluated for both vibration measurements, with particular attention to their ability to reflect progressive crack growth.
4. (4) The principal trends obtained from the numerical comparison were further examined experimentally using a separate fixed-shaft spur gearbox.

2. Dynamic Modeling

The objective of dynamic modeling of the system is to extract the transverse and torsional vibrations for various crack sizes. This modeling involves calculating the mesh stiffness of the engaged gear teeth and deriving the system equations.

2.1. Mesh Stiffness of Gear Tooth

The most critical part of system modeling is the calculation of the mesh stiffness between paired gear teeth, as a principal source of vibration is tooth engagement and impact. Methods for calculating gear tooth mesh stiffness include the potential energy method, the square wave method, experimental methods and the finite element method. Among these, the potential energy method offers sufficient computational accuracy while maintaining relatively low computational cost. In this section, the time-varying mesh stiffness of the gear tooth is calculated using the potential energy method, which models each tooth as a cantilever beam. The mesh stiffness of gears during single tooth engagement can be expressed by Equation (1) [18]:

$$k_t = \frac{1}{\left(\frac{1}{k_{b1}} + \frac{1}{k_{s1}} + \frac{1}{k_{a1}} + \frac{1}{k_{b2}} + \frac{1}{k_{s2}} + \frac{1}{k_{a2}} + \frac{1}{k_h}\right)} \quad (1)$$

where k_t is the mesh stiffness between the gears and k_b , k_s , k_a and k_h represent the bending, shear, axial compression and Hertzian contact stiffness, respectively. The subscripts 1 and 2 refer to the driving and driven gears, respectively. To calculate the mesh stiffness of the gears when two pairs of teeth are engaged, Equation (2) is used [18]:

$$k_t = \sum_{i=1}^2 \frac{1}{\left(\frac{1}{k_{b1,i}} + \frac{1}{k_{s1,i}} + \frac{1}{k_{a1,i}} + \frac{1}{k_{b2,i}} + \frac{1}{k_{s2,i}} + \frac{1}{k_{a2,i}} + \frac{1}{k_{hi}}\right)} \quad (2)$$

in which $i = 1$ refers to the first pair of teeth, and $i = 2$ refers to the second pair of teeth.

Considering that in the potential energy method, the tooth is modeled as a cantilever beam, when a local defect occurs in the gear tooth, the effective length and thickness of the tooth change, resulting in a reduction of the overall mesh stiffness. For a standard spur gear with a pressure angle of 20 degrees, when the number of teeth

is less than 41, the root circle is smaller than the base circle and when the number of teeth is greater than 41, the root circle is larger than the base circle [18].

Complete formulas for calculating the mesh stiffness of healthy and defective gears for both cases (when the number of teeth is less than 41 and when it is greater than 41) have been provided in reference [18]. In this study, the crack geometry was defined consistently with the reference model in Ref. [19] which was used for validation of the numerical results. As shown in Figure 1, the crack growth on the tooth starts from point M located on the root circle and extends at an angle ν relative to the central line of tooth up to point B on the central line. This point represents 50% of the crack length. With further crack growth from point B to point D, representing 100% of the crack length, the tooth completely separates from the gear. Similarly, cracks smaller than 50%, which were used in this study, were expressed as a fraction of the length MB. The $\nu=45^\circ$ crack orientation was adopted to maintain consistency between the crack geometry used in the reference model, the present numerical simulations, and the experimental crack configuration.

In this study, the mesh stiffness was calculated for the values provided in Table 1. Figure 2 shows the mesh stiffness results for one and two pairs of gear teeth over a complete mesh cycle of defective tooth for various crack sizes. As evident from Figure 2, as the crack in the tooth grows, the mesh stiffness of the defective tooth decreases both when two pairs of teeth are in contact and when only one pair of teeth is in contact.

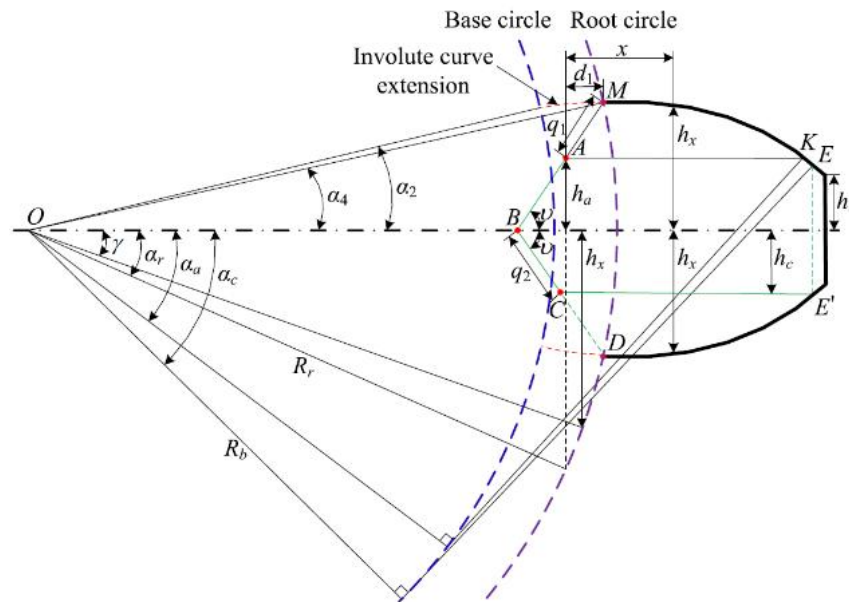


Figure 1. Cracked tooth for the case where the root circle is larger than the base circle [18]

Table 1. Gearbox properties

	Tooth number	Modulus (mm)	Pressure angle (deg)	Mass moment of inertia (kg.mm ²)	Mass (kg)	Young's modulus (GN/m ²)	Poisson ratio	Rotation frequency (Hz)	Width (mm)
Pinion	55	2	20	1247.9	1.53	200	0.3	16.6	20
Gear	75			2338.8	3.01			12.2	
	Stiffness			Damping					
Bearing	200 MN/m			10 kN.s/m					
Shaft	50 MN.m/rad			10 kN.s/rad					

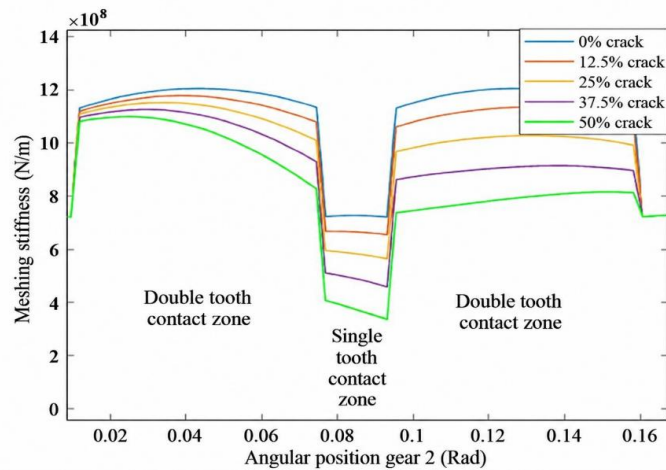


Figure 2. Gear tooth mesh stiffness for various crack sizes

2.2. Dynamic Equations of Transverse and Torsional Vibrations

In this study, to model the dynamic behavior of a single-stage simple gearbox shown in Figure 3, the lumped parameter method was used. A reduced-order 6-DOF formulation was selected because the objective of the present study is not to reproduce all structural modes of the gearbox, but to provide a consistent dynamic framework for directly comparing the crack-induced transverse and torsional responses. The model includes the

transverse motions of the pinion and gear and the rotational motions of the pinion, gear, motor, and load, thereby retaining the principal degrees of freedom required for the two response quantities investigated in this work. Effects such as housing flexibility, multiple shaft bending modes, gyroscopic coupling, and other higher-order structural dynamics are not explicitly represented and are considered outside the scope of the present reduced-order comparison. Accordingly, the adequacy of the present model is evaluated with respect to its intended comparative purpose rather than its ability to reproduce the complete structural dynamics of a practical gearbox. The gears, motor, and brake were modeled as disks with inertia undergoing rotational motion. The system has six degrees of freedom, including the rotation of pinion θ_p , the rotation of gear θ_g , the rotation of the motor θ_m , and the rotation of the load (brake) θ_L , along with transverse displacement in one direction for pinion y_p and gear y_g .

These degrees of freedom are illustrated in Figure 3, and the system's differential equations, based on the considered degrees of freedom (which are the unknown variables), are expressed by Equation 3 [10].

$$\begin{cases} I_p \ddot{\theta}_p + C_{t1}(\dot{\theta}_p - \dot{\theta}_m) + k_{t1}(\theta_p - \theta_m) = -Wr_p \\ I_g \ddot{\theta}_g + C_{t2}(\dot{\theta}_g - \dot{\theta}_L) + k_{t2}(\theta_g - \theta_L) = -Wr_g \\ I_m \ddot{\theta}_m + C_{t1}(\dot{\theta}_m - \dot{\theta}_p) + k_{t1}(\theta_m - \theta_p) = T_m \\ I_L \ddot{\theta}_L + C_{t2}(\dot{\theta}_L - \dot{\theta}_g) + k_{t2}(\theta_L - \theta_g) = -T_L \\ m_p \ddot{\theta}_p + C_{yp} \dot{y}_p + k_{yp} y_p = W \\ m_g \ddot{\theta}_g + C_{yg} \dot{y}_g + k_{yg} y_g = -W \end{cases} \quad (3)$$

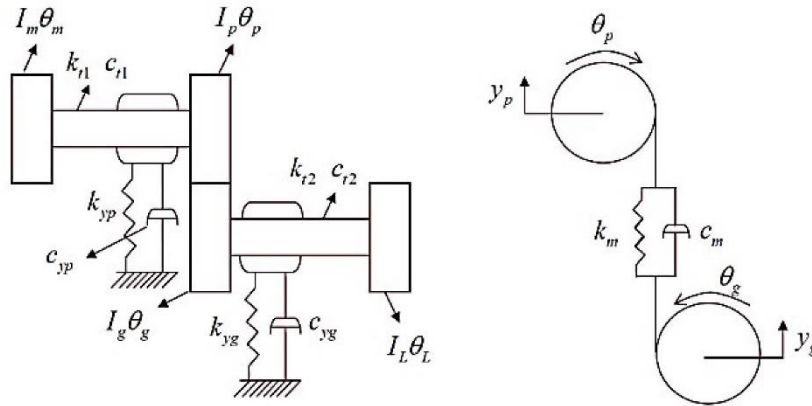


Figure 3. The 6 degree-of-freedom gearbox model [10]

The dynamic force W was obtained from Equation 4, and the time-varying mesh damping was derived from Equation 5 [10].

$$W = k_m(\theta_p r_p - \theta_g r_g + y_g - y_p) + c_m(\dot{\theta}_p r_p - \dot{\theta}_g r_g + \dot{y}_g - \dot{y}_p) \quad (4)$$

$$c_m = 2\zeta_m \sqrt{k_m m_p m_g / (m_p + m_g)} \quad (5)$$

where ζ represents the damping ratio, which was set to 0.07 according to reference [18]. Other parameters used in the system's differential equations are defined as follows:

- T_m, T_L : Motor torque, load torque.
- I_m, I_L, I_p, I_g : Mass moments of inertia of the motor, load, pinion and gear.
- $\theta_m, \theta_L, \theta_p, \theta_g$: Angular displacements of the motor, load, pinion and gear
- k_{t1}, k_{t2} : Torsional stiffness of the first shaft (between pinion and motor) and the second shaft (between gear and load)
- C_{t1}, C_{t2} : Torsional damping of the first shaft (between pinion and motor) and the second shaft (between gear and load)
- k_{yp}, k_{yg} : Bearing stiffness of the pinion and gear
- C_{yp}, C_{yg} : Bearing damping of the pinion and gear
- m_p, m_g : Mass of the pinion and gear
- y_p, y_g : Vertical displacements of the pinion and gear bearings
- k_m, c_m : Time-varying mesh stiffness and mesh damping
- r_p, r_g : Base circle radius of the pinion and gear

The motor torque was obtained from Equation 6.

$$T_m = \frac{3sE^2R}{2\pi n_s(R^2 + (sX)^2)} \quad (6)$$

in which R and X are the rotor impedance components, E is the motor voltage, and n_s is the synchronous speed of the motor obtained from Equation 7. The slip, s , was calculated from Equation 8.

$$n_s = 2f/p \quad (7)$$

$$s = 1 - \dot{\theta}_m/n_s \quad (8)$$

where f is the frequency and p is the number of poles of the electric motor.

3. Numerical Results and Discussion

Based on the gearbox properties listed in Table 1 and the electric motor specifications in Table 2, the system's differential equations were solved using the Runge-Kutta numerical method via the ode15s solver in MATLAB. To use this solver, the six second-order differential equations of the system must be converted into twelve first-order differential equations. The initial conditions for all twelve variables, including the degrees of freedom and their derivatives, were set to zero, and the load torque was considered to be 11 Newton-meters. For vibration analysis, transient responses were ignored, and only steady-state conditions were considered. The transverse dynamic response considered in this study is represented by the acceleration signal of the pinion, whereas the torsional dynamic response is characterized by the dynamic torque of the pinion shaft. It should be noted that dynamic torque and angular displacement or angular velocity are distinct physical quantities. However, the dynamic torque transmitted through a shaft is directly related to the relative torsional deformation and relative angular velocity of the connected components through the torsional stiffness and damping of the shaft. Therefore, the dynamic shaft-torque signal provides a physically meaningful representation of the torsional dynamic response associated with crack-induced variations in gear mesh stiffness.

To allow for validation of the results later, parameter values and system specifications from reference [19] were initially used. Then, the motor specifications were selected to simulate the same gearbox speed as in the reference paper, which is 1000 rpm. It is worth mentioning that in reference [19], only the acceleration signal results were extracted.

Table 2. Motor specifications

E (volt)	R (Ω)	X (Ω)	f (Hz)	<i>No. of poles</i>
220	0.2409	0.5406	50	6

Figure 4 and Figure 5, respectively, show the simulated acceleration and torque signals. As observed, when the cracked tooth engages, the waveform of both vibration signals changes in amplitude, and this change corresponds to the crack length. Figure 6 shows the acceleration signal results from reference [19]. The comparison with the published transverse acceleration response in reference [19] provides a qualitative check of the selected reduced-order model. The simulated response exhibits a comparable order of magnitude and a similar dependence on crack size. This comparison is not intended as a formal convergence or model-fidelity study; rather, it supports the use of the 6-DOF formulation as a reduced-order framework for the comparative analysis of transverse and shaft-torsional responses conducted in this work. The differences in the waveform shapes are mainly due to discrepancies in the mesh stiffness results between this study and reference [19].

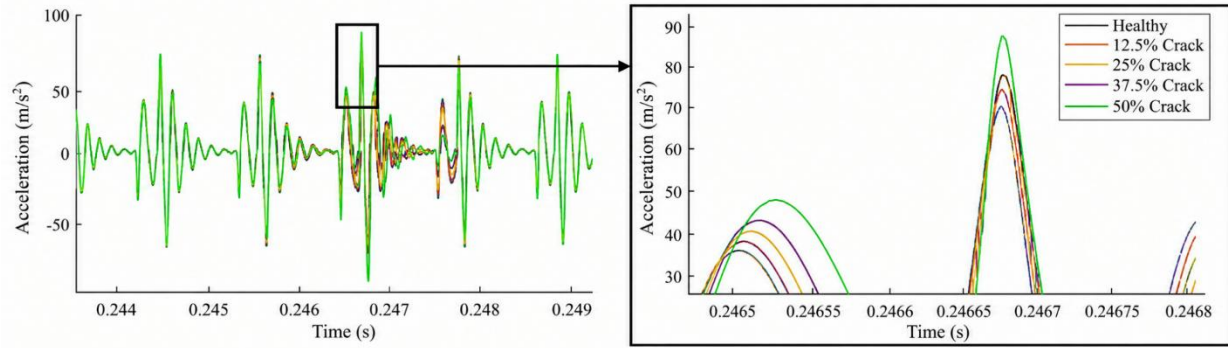


Figure 4. The waveform of the gearbox transverse acceleration signal for different crack sizes

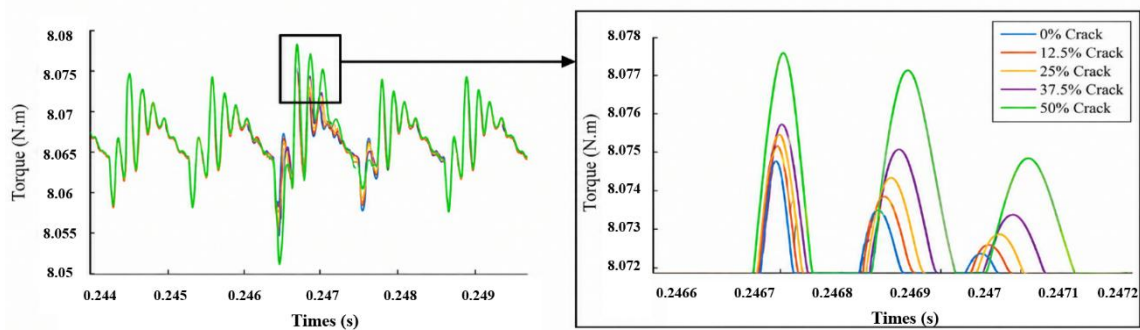


Figure 5. The waveform of the gearbox torque signal for different crack sizes

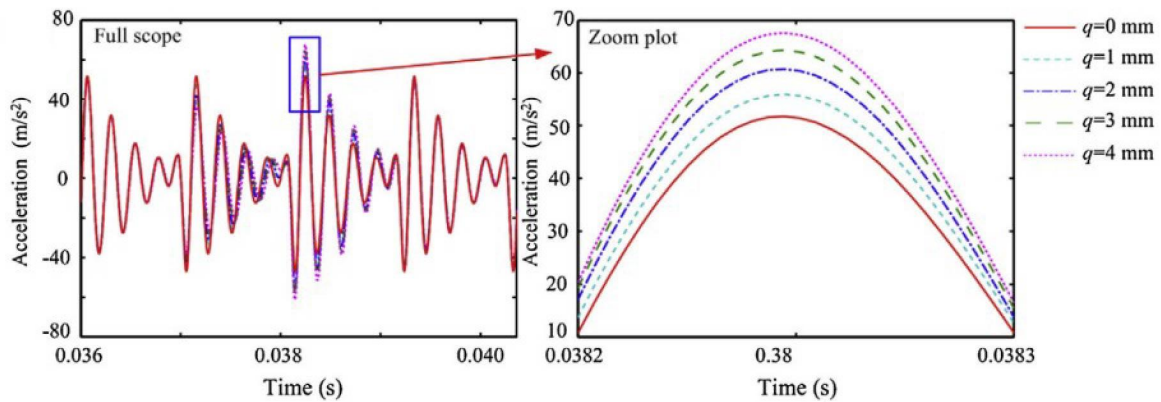


Figure 6. The waveform of the transverse acceleration signal for different crack sizes [19]

The simulated transverse and torsional vibration signals are analyzed using statistical indicators in the time domain. The statistical indicators include the root mean square (RMS), kurtosis, and crest factor, which are defined by equations 9 to 11 [10, 19]. These indicators are among the most common metrics used in machine fault diagnosis through vibration analysis, where an increase in their values typically indicates the progression of a fault in the system.

$$\text{RMS} = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2} \quad (9)$$

$$\text{kurtosis} = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^4}{\left(\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \right)^2} \quad (10)$$

$$\text{Crest Factor} = \frac{\text{signal peak magnitude}}{\text{RMS}} \quad (11)$$

where x_i , $\bar{x}$, and n represent the i -th component of the signal, the mean of the signal and the length of the signal, respectively. Figure 7 shows the statistical indicators extracted from the reference [19] along with the simulated transverse vibration signals in this work. A crack size of 4 mm represents 50% crack, where all index variations are calculated as percentage changes relative to the healthy (uncracked) condition. The percentage changes in both the RMS and kurtosis indices agree with those reported in reference [19] within the same order of magnitude, with the reasons for their differences explained above. Figure 8 additionally presents the statistical indices of torsional vibration signals for each crack size, along with the transverse vibration data, all compared to the healthy state. As the crack propagates, these indicators increase for both the acceleration and torque signals. However, the torsional vibration, represented by the dynamic torque signal exhibited greater sensitivity to fault growth than the transverse vibration represented by the acceleration signal. For example, in the case of a 50% crack, the kurtosis of the torque signal increases by 6.8%, while the acceleration signal shows only 1% increase. Additionally, the crest factor appears to be more effective for early-stage crack detection compared to other indicators.

Its more linear trend allows for a clearer indication of fault progression; a 10% variation in the crest factor at 12.5% crack propagation is sufficient to confirm crack growth.

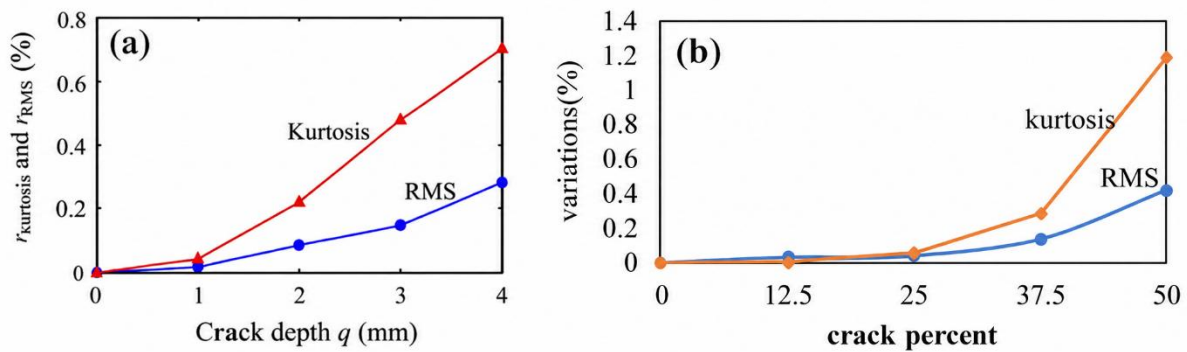


Figure 7. Changes in kurtosis and root mean square (RMS) of gearbox acceleration signal with 0% to 50% crack sizes (a) reference [19] (b) in this paper

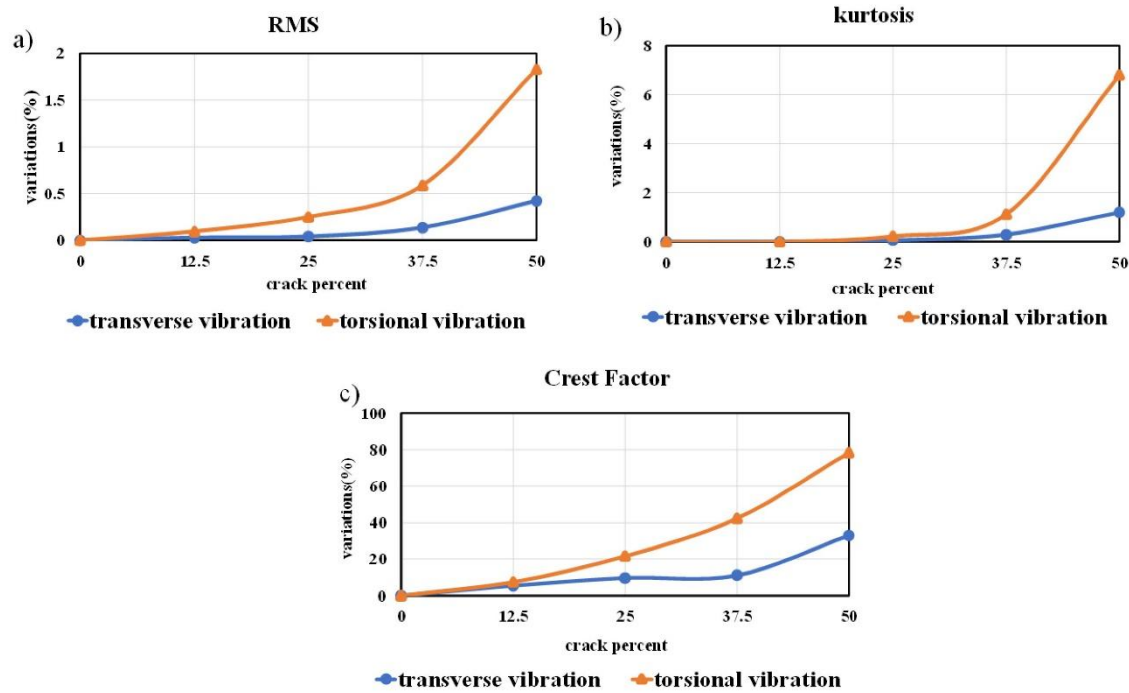


Figure 8. Change in statistical indicators relative to the healthy state for transverse and torsional vibrations

The difference in sensitivity of the transverse and shaft-torsional responses can be interpreted in terms of the propagation of crack-induced mesh excitation through the drivetrain, as conceptually illustrated in Figure 9. A tooth-root crack reduces the local mesh stiffness and consequently modifies the mesh force and the transmitted torque during the engagement of the defective tooth.

These crack-induced variations are coupled to the torsional dynamics of the drivetrain and are reflected in the relative torsional deformation and torque fluctuations of the connected shafts. In contrast, as shown schematically in Figure 9, the transverse acceleration measured on the gearbox structure represents a response that is additionally shaped by the dynamic characteristics of the gears, bearings, housing, structural mounting, and sensor location. Consequently, the measured transverse response may be influenced by structural transmission and filtering effects in addition to the crack-induced variation in mesh stiffness. In the present configuration, this difference in dynamic transmission is reflected in the larger relative changes observed in the shaft-torsional response. However, this result should be interpreted as configuration-dependent rather than as a universal property of torsional measurements. The following experimental study was therefore used to examine whether a similar sensitivity trend can also be observed in a separate fixed-shaft gearbox configuration.

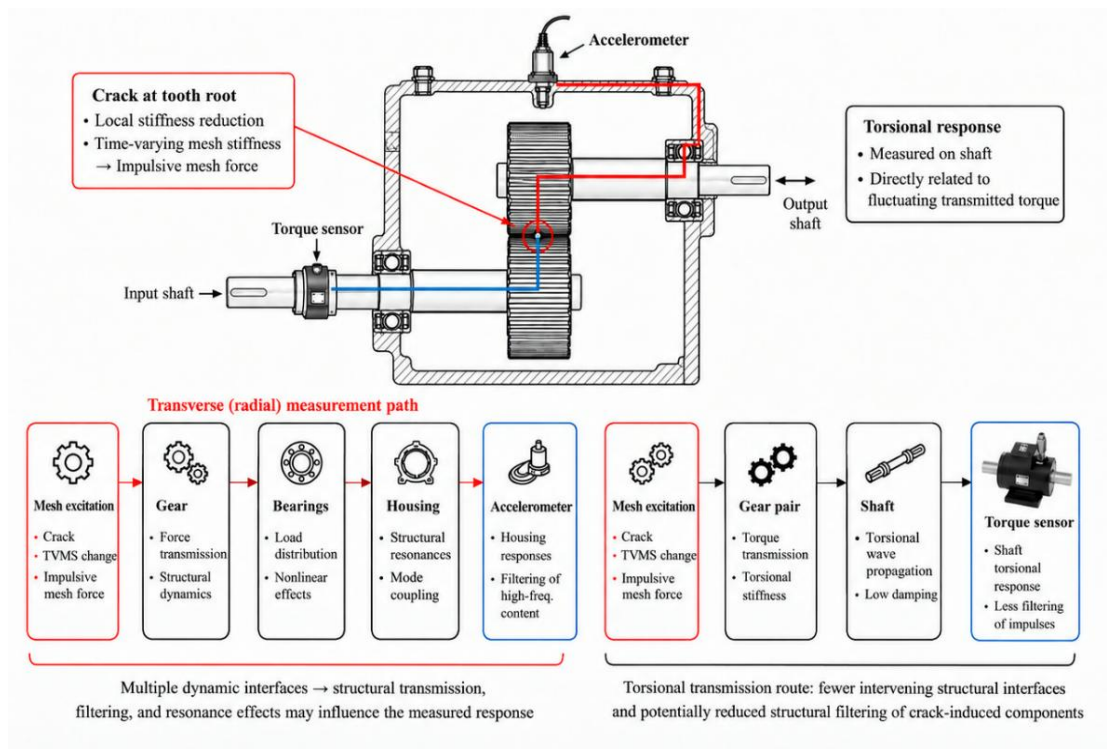


Figure 9. Conceptual illustration of the transmission paths of crack-induced gear-mesh excitation to the transverse and shaft-torsional measurement responses.

4. Experimental Investigation

In this section, the explanation of the experimental setup and analysis of the measured results are presented.

4.1. Laboratory Setup

To experimentally examine the principal trend observed in the numerical analysis, the experimental setup shown in Figure 10 was employed. In this setup, a single-stage spur-gear gearbox with a gear ratio of 1:1 was driven by an electric motor. A magnetic powder brake was connected to the gearbox output shaft to apply load. An accelerometer was mounted on the gearbox housing in the vertical direction to measure transverse vibrations, while a dynamic torque transducer was installed along the shaft to characterize the torsional vibrations

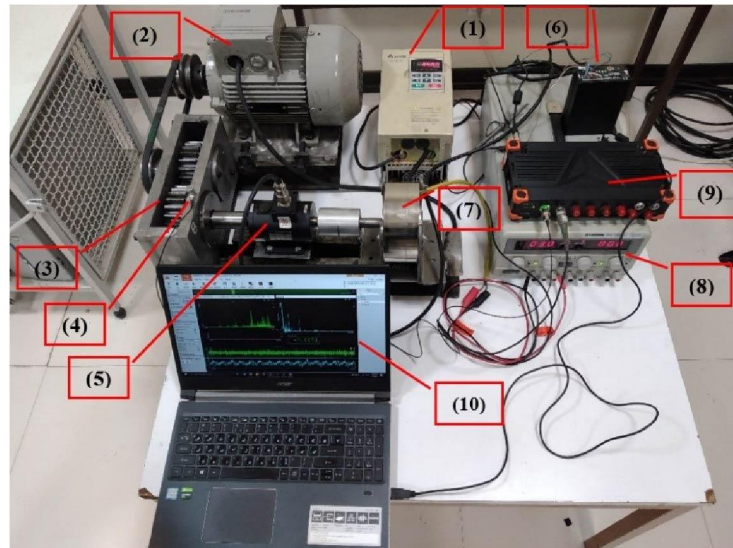


Figure 10. Experimental setup: (1) inverter, (2) electromotor, (3) spur gearbox, (4) accelerometer, (5) torque sensor, (6) amplifier, (7) magnetic powder brake, (8) brake driver, (9) DAQ, (10) DAQ analyzer

To simulate a fault, a crack was manually introduced at the root of one tooth on the output gear of the gearbox with the same 45° orientation used in the numerical model and in the reference study [19]. The crack spanned the full width of the tooth and was oriented at a 45° -degree angle relative to the tooth centerline. During each stage of the test, the crack length was progressively increased until tooth fracture occurred (as shown in Figure 11).

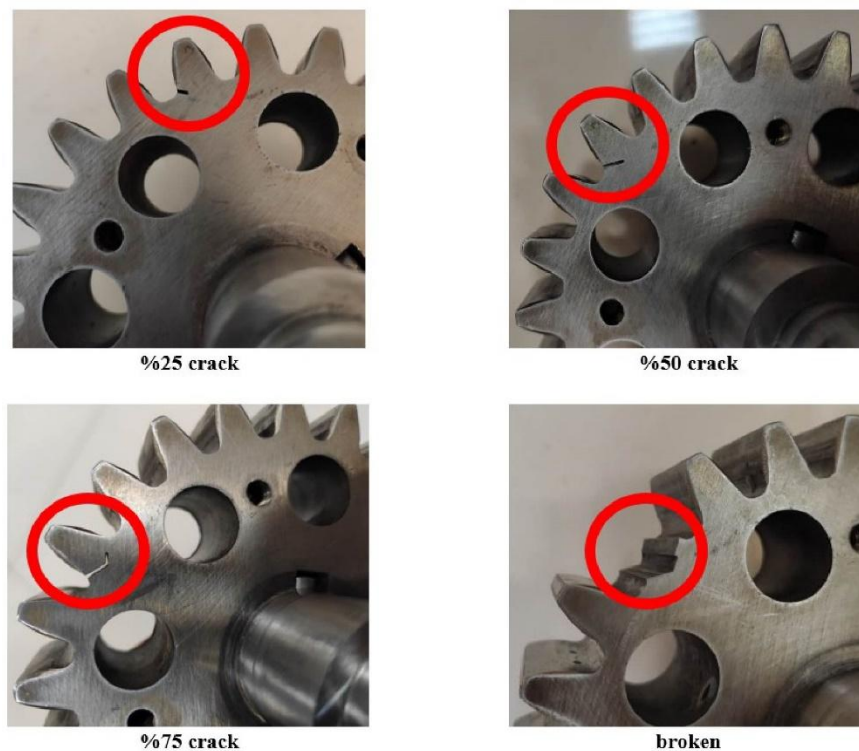


Figure 11. Crack defect in the gear tooth

4.2. Experimental Signal Analysis

The sampling frequency of the measured transverse and torsional vibration signals was 6000 Hz. The signals of various crack sizes are presented in Figure 12. First, the transverse and torsional vibrations were measured while the gearbox was in a healthy condition, and the values of the statistical indicators—RMS, kurtosis, and crest factor—were calculated. Then, for each level of defect growth, the statistical indicators are recalculated, and the changes in each indicator relative to the healthy state were determined.

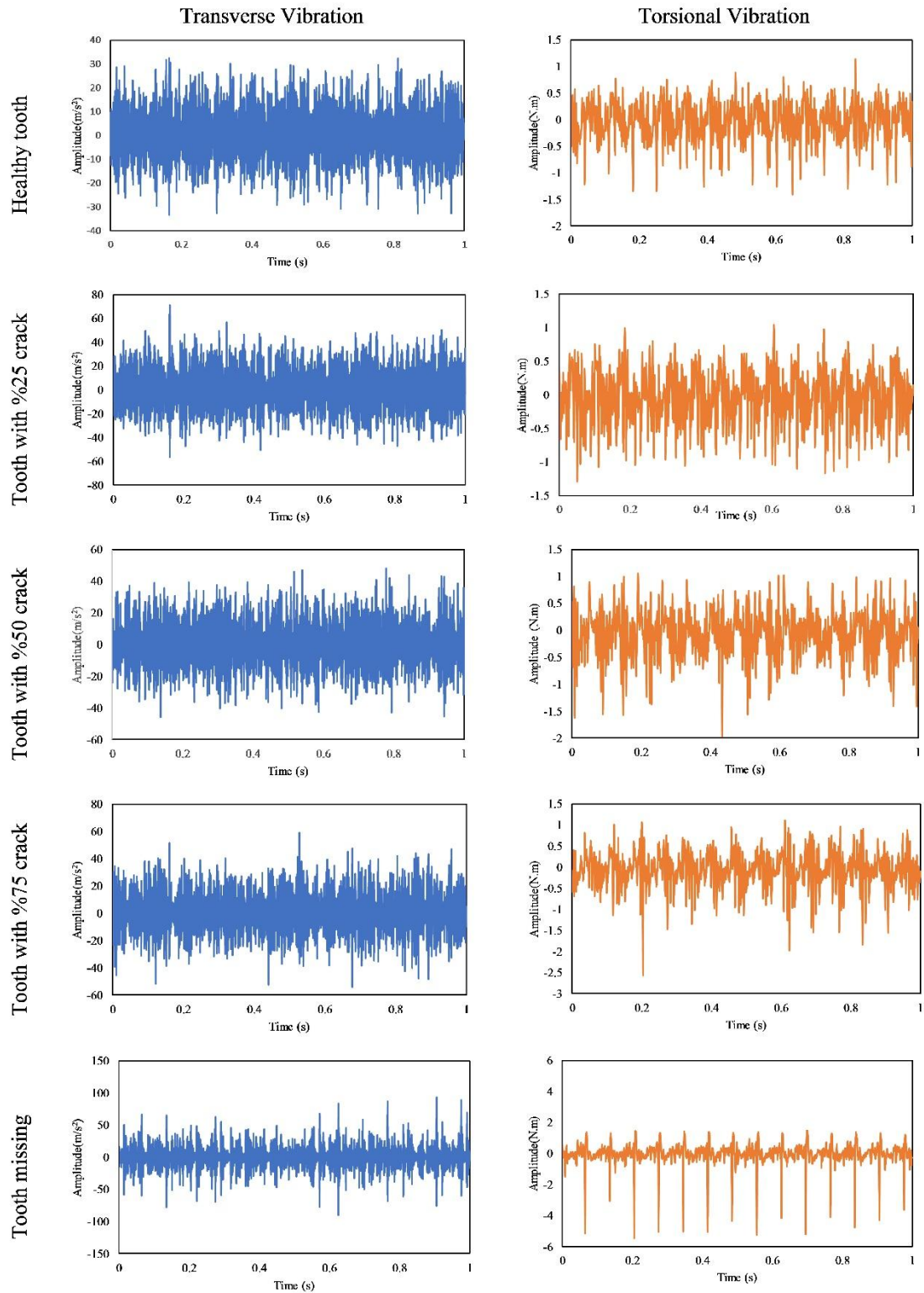


Figure 12. Measured signals from experimental setup

Figure 13 illustrates the changes in various statistical indicators, including RMS, kurtosis, and crest factor, for both transverse and torsional vibrations across different fault conditions: 25%, 50%, and 75% crack sizes, and

complete tooth breakage, all relative to a healthy tooth. As shown in Figure 13(a), the transverse RMS value at the 25% crack level is unexpectedly higher than those at subsequent crack stages. While this could potentially be attributed to installation-related errors, the lack of repeat measurements prevents a conclusive determination of its source; therefore, this observation is treated as an experimental deviation. Furthermore, it highlights that RMS, on its own, does not provide a consistently monotonic measure of crack progression within the scope of this experiment. In Figure 13(b), the kurtosis changes for transverse vibration at 25%, 50%, 75% crack growth, and tooth loss are 9%, 5%, 18%, and 111%, respectively, when compared to the healthy tooth. For torsional vibration, the corresponding changes are -13%, 30%, 47%, and 492%, respectively. The negative variation at the 25% crack level indicates that torsional kurtosis does not exhibit a strictly monotonic response during the earliest stage of crack growth. At this relatively small crack level, the fault-induced impulsive component may still be insufficiently dominant relative to the overall system response and measurement variability, causing kurtosis to decrease rather than increase.

However, from the 50% crack level onward, torsional kurtosis shows a clear increase and becomes substantially more responsive to crack growth than the corresponding transverse vibration.

Therefore, kurtosis should not be interpreted as a universally monotonic crack-growth indicator at all fault stages, whereas its overall response at moderate and advanced crack levels demonstrates the higher sensitivity of torsional vibration. In Figure 13(c), the crest factor provides a clearer representation of crack progression. For transverse vibration, the changes at 25%, 50%, 75% crack growth, and complete tooth loss are 5%, 8%, 7%, and 55%, respectively, showing only limited variation before complete tooth failure. In contrast, the torsional vibration exhibits a more consistent increase, with corresponding changes of 5%, 42%, 74%, and 117%.

These experimental results suggest that the dynamic shaft-torque response may exhibit higher sensitivity to fault progression than the transverse acceleration response when appropriate response indicators are selected. Among the indicators examined, the shaft-torque crest factor provided the clearest experimental evidence of progressive crack growth.

The torsional vibration exhibits higher sensitivity to fault progression and offers better early fault detection than the transverse vibration. This difference can be interpreted in terms of the different dynamic transmission mechanisms of the two measured responses. Crack-induced variations in mesh stiffness directly affect the transmitted mesh force and shaft torque, whereas the transverse acceleration measured on the gearbox structure is additionally influenced by the dynamic characteristics and transmission effects of the gears, bearings, housing, mounting structure, and sensor location. These findings are consistent with the numerical results and further support the comparative advantages of torsional vibration signals. Among the indicators examined in this study, the torsional crest factor therefore provides the clearest experimental evidence of increasing sensitivity to progressive crack growth.

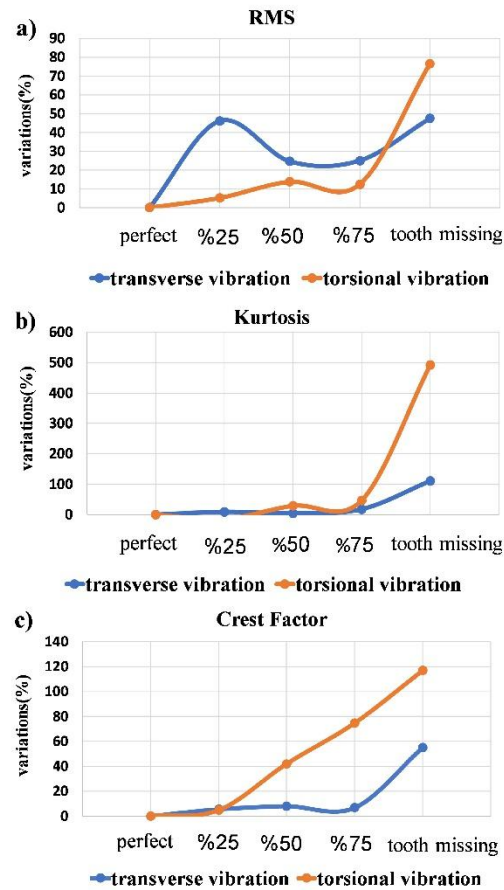


Figure 13. Variations of each statistical indicator for different crack sizes

5. Conclusions

This study aimed to compare the sensitivities of transverse and torsional vibrations to progressive gear tooth crack growth in a single-stage fixed-shaft spur gearbox. Unlike previous torsional-vibration studies on planetary gear systems, where the principal advantage of torsional measurements is often associated with the absence of time-varying transmission-path modulation, the present study shows that the torsional response can provide higher sensitivity to crack growth even when this planetary-gear-specific modulation mechanism is absent. The dynamic behavior of the power transmission system was numerically simulated using a lumped parameter method, considering the time-varying mesh stiffness of the gear teeth for both healthy and faulty conditions. Analysis of these simulations using time-domain statistical indicators such as RMS, kurtosis, and crest factor, showed that the dynamic shaft-torque response was generally more sensitive to crack progression than the transverse acceleration response for the investigated configuration. The relative sensitivity depended on the selected statistical indicator, with the crest factor showing particularly clear changes at the early stages of crack growth. The numerical findings were subsequently examined experimentally using a separate single-stage fixed-shaft spur gearbox. Although the experimental gearbox had a different gear ratio and dynamic characteristics from the numerical model, the experimental investigation was designed to examine whether the principal trend predicted by the numerical analysis could also be observed under a consistent crack geometry. The experimental results supported the observed trend of higher torsional-vibration sensitivity to progressive crack growth, particularly at moderate and advanced crack levels.

Therefore, the experimental results are regarded as experimental support for the main numerical finding rather than as a strict quantitative validation of the complete numerical model. The higher sensitivity of torsional vibration observed in both the numerical and experimental results can be attributed to the different dynamic transmission mechanisms of the two responses. Crack-induced changes in gear-mesh excitation are directly coupled to the torsional dynamics of the drivetrain, whereas the transverse vibration measured on the gearbox housing is additionally affected by the structural transmission characteristics of the gears, bearings, housing, mounting conditions, and sensor location. Furthermore, both sets of results highlighted the crest factor as a more effective indicator for early-stage crack detection compared to other analyzed statistical indicators, due to its consistent and discernible change with the onset of a fault.

From a practical condition-monitoring perspective, the higher sensitivity observed for the dynamic shaft-torque response should be balanced against the additional requirements associated with its measurement. Compared with accelerometers mounted on stationary gearbox housings, dynamic torque transducers generally require integration into the rotating drivetrain and may involve additional mechanical, installation, calibration, and instrumentation requirements. Consequently, the higher diagnostic sensitivity observed for the shaft-torque response should be considered together with the cost and practical feasibility of the measurement system. Therefore, the present study does not suggest that dynamic torque measurement should universally replace conventional accelerometer-based monitoring; rather, it indicates that, when technically and economically feasible, dynamic shaft-torque measurement can provide a complementary and potentially more sensitive response for monitoring progressive gear tooth cracking.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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