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REVIEW PAPER

Review on Dynamic Modeling and Vibration Characteristics of Rotating Cracked Blades

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***Abstract**:* As one of the most important parts in the engine, the structure and state of the rotating blade directly affect the normal performance of the aeroengine. In order to monitor engine crack failure and ensure flight safety, it is necessary to carry out research on the dynamic modeling of the cracked blade and breathing crack-induced vibration mechanisms. This paper summarizes the current research status on the dynamics of cracked blade, and the related topics mainly include four aspects: crack propagation path, mechanical model of open and breathing cracks, dynamic modeling methods of cracked blades such as lumped mass model, semi-analytical model and finite element model, and dynamic characteristics of cracked blades. The review will provide valuable references for future studies on dynamics and fault diagnosis of cracked blade in aeroengine.

***Keywords**:* breathing crack; crack propagation; cracked blade; dynamic characteristics; dynamic modeling

I. INTRODUCTION

Blade is one of the core components of rotating machinery such as aeroengines, gas turbines, and axial flow compressors. With the development of aeroengine technology and the improvement of performance requirements, blades operate under extremely harsh environments such as high speed, high temperature, heavy load, and strong disturbance, leading to more and more serious failure problems related to blade cracks as shown in Fig. 1. These failures seriously affect the safety and economy of rotating machines. It is necessary to study crack-induced vibration mechanism, dynamic characteristics, and typical features so as to provide theoretical support for the vibration monitoring of the blade and crack fault diagnosis.

Many researchers have carried out a lot of research on crack propagation path, crack mechanical model, cracked blade dynamic modeling, dynamic characteristics, and crack diagnosis [2–9]. The crack propagation research mainly analyzes the crack propagation law under different initial damage and load conditions by the method of fracture mechanics and mainly provides an accurate path for the subsequent crack mechanics model. Cracks are mainly described by open crack and breathing crack, and breathing crack can describe the time-varying effect of crack stiffness with load change. The dynamic modeling of cracked blades is mainly based on lumped mass method (LMM), semi-analytical method (SAM), and finite element (FE) method. The influence of open or breathing crack on natural characteristics and vibration response characteristics can be analyzed, and the influence mechanism between cracks and system dynamics characteristics can be revealed. The cracks are qualitatively or quantitatively diagnosed by the change of natural characteristics and nonlinear vibration response.

In this paper, the recent research progress on the prediction of crack propagation path, crack mechanics model, dynamic modeling method of cracked blade, and dynamic characteristics is summarized. The overall structure of this paper is as follows. In Section II, the prediction methods of crack growth path is introduced. In Section III, the mechanical models of open crack and breathing crack are reviewed. Sections IV and V mainly introduce the dynamic modeling methods and dynamic characteristics of rotating cracked blade. The conclusions and future prospects are given in Section VI. The relationship between the main contents is shown in Fig. 2.

II. PREDICTION METHODS ON THE CRACK PROPAGATION PATHS

In recent years, the numerical simulation of fatigue crack propagation path by computer has become a reality. At present, the most commonly used numerical calculation methods include traditional FE method (TFEM), extended

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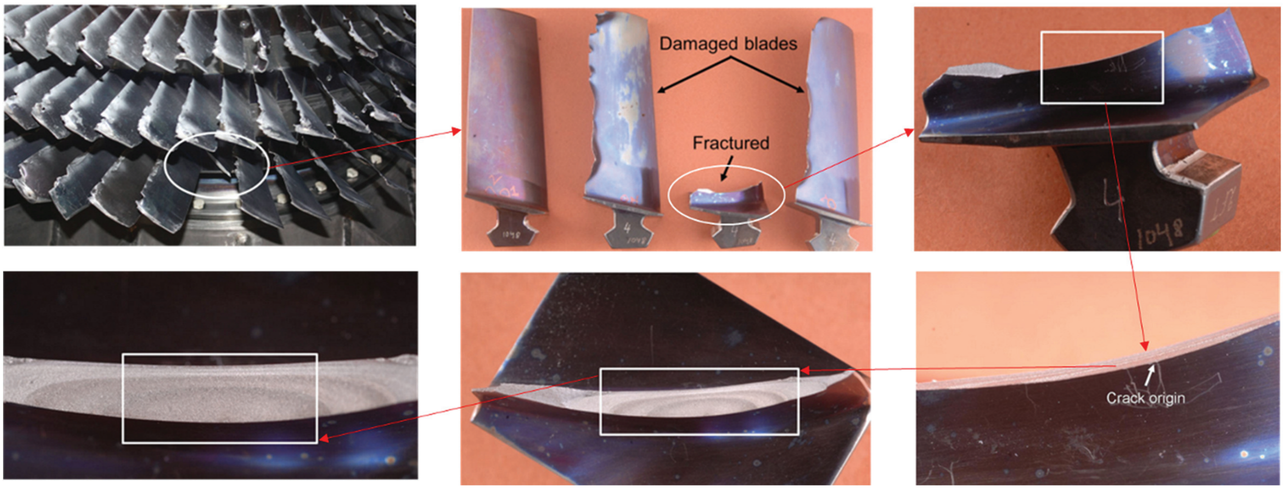


Fig. 1. Blade fracture in the compressor section of the aeroengine [1].

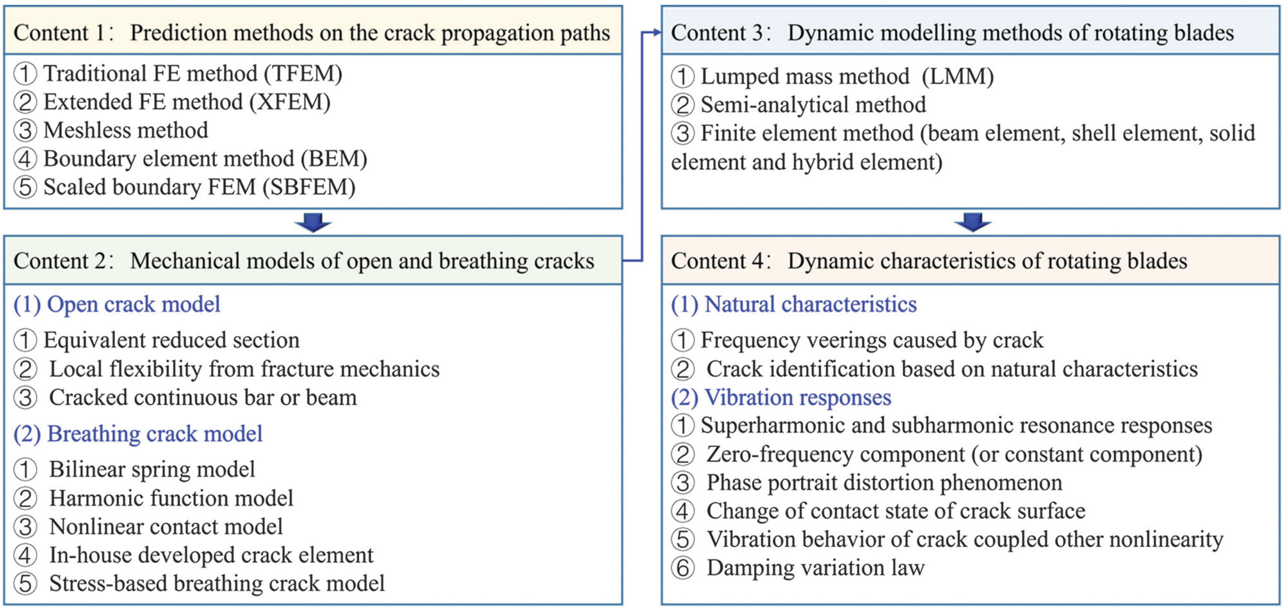


Fig. 2. The relationship between the main contents.

FEM (XFEM), meshless method, finite difference method (DFM), boundary element method (BEM), and FEM-BEM. The common software used for crack propagation simulation mainly includes Franc3D, Zencrack, Beasy, etc.

A. TRADITIONAL FE METHOD

TFEM is suitable for arbitrary geometric shapes and boundary conditions, material and geometric nonlinear problems, anisotropy problems, and easy programming, so it has become the main method for simulating discontinuous problems, such as cracks. There are two methods to simulate the crack propagation in FE analysis, i.e., method 1: local crack tip and wake remeshing and method 2: birth-death element [10].

For method 1, the refined mesh is usually adopted at the crack tip and the possible crack propagation area, which greatly reduces the solution efficiency. At present, many commercial FE software (Ansys, Abaqus, etc.) combined with fracture mechanics tools (Franc3D, Zencrack, etc.) are used for predicting crack growth and fatigue life [11–13]. Mangardich *et al.* [11] established the FE model of aircraft engine fan dovetail attachments using Ansys software and adopted Franc3D software to carry out crack propagation simulation using the maximum tensile stress theory, as shown in Fig. 3. Based on the similar method using Ansys and Franc3D, they also carried out a 3D crack propagation simulation to simulate the fracture process of an aircraft engine high pressure compressor blade, as shown in Fig. 4 [12]. Shlyannikov and Ishtyryakov [14] derived a crack propagation rate equation considering the effects of the fracture zone size and nonlinear stress intensity factor (SIF) and analyzed the crack growth process of an aviation gas turbine engine compressor disk. Combined with Abaqus and Franc3D, Poursaeidi and Bakhtiari [15] performed the crack propagation simulation for the first stage of compressor blade. Aiming at turbine blades, Liu *et al.* [16] used Franc3D for remeshing, SIF calculation, and

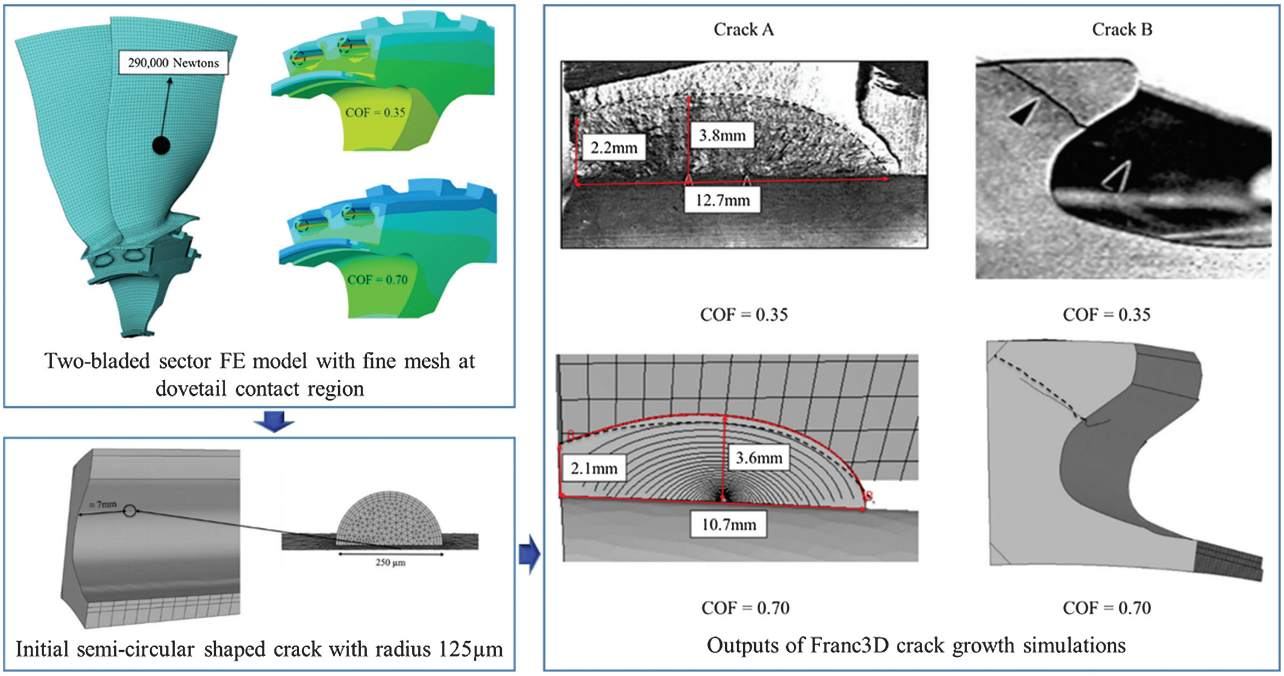


Fig. 3. Crack propagation simulation of aircraft engine fan dovetail attachments using Ansys and Franc3D software [11].

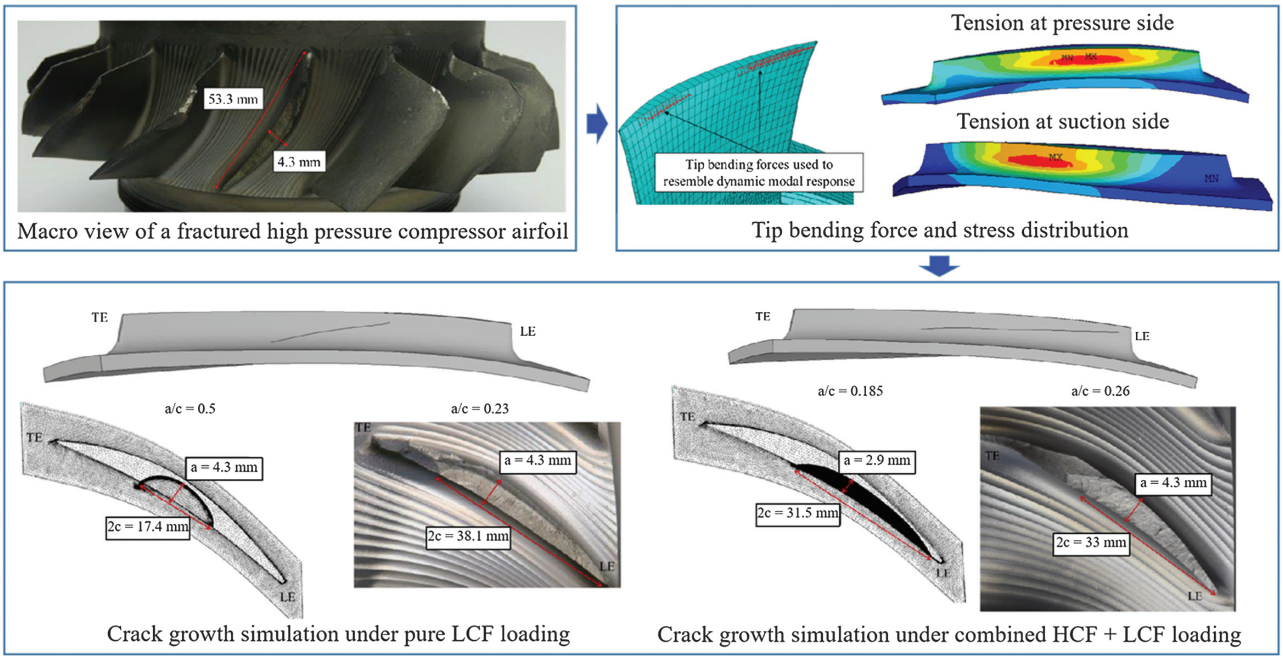


Fig. 4. Crack propagation simulation of an aircraft engine high pressure compressor blade using Ansys and Franc3D software [12].

crack propagation implementation and carried out stress updating after crack propagation in Abaqus. Canale *et al.* [17] studied the low-cycle fatigue crack growth process of an aeroengine blade structure by recalculating the stress field as a crack propagation function using Franc3D. Salehnasab and Poursaeidi [18] studied the crack initiation and growth of a blade in gas turbine engine using Abaqus and Zencrack software, which adopt J-integral and crack tip opening displacement to calculate the SIF and the maximum energy release rate criterion to determine the crack propagation direction. Wang *et al.* [19] simulated the surface crack propagation process of an airfoil section blade under aerodynamic and centrifugal loads by Ansys and Franc3D, and the maximum circumferential stress criterion was used to determine the crack propagation direction.

For method 2, a moving template procedure is developed to follow the crack tip physics in a self-adaptive way, which can provide an accurate crack propagation prediction. This method adopts the global model, the interpolatively tied moving template, and a multilevel element death option to simulate the crack growth process [20,21].

B. EXTENDED FEM

XFEM improves the deficiency of TFEM according to the idea of partition of unity and allows the element discontinuities by adding degrees of freedom (DOFs) with special displacement functions [22–24]. Hu *et al.* [22] adopted the crack propagation path of a dovetail assembly by XFEM, which adopts the maximum tangential stress criterion and the maximum energy release rate criterion to predict the crack propagation direction. Considering the effects of upstream distortion, Dompierre *et al.* [23] adopted XFEM implemented in Samcef software to study the crack propagation of a fan blade by assuming the crack initiation in the maximum principal stress region, as shown in Fig. 5. Holl *et al.* [24] carried out 3D crack growth simulation for a gas turbine blade using the XFEM, which adopts J-integral to calculate SIFs and the criterion of maximum hoop stress criterion to predict a precise propagation direction. Bergara *et al.* [25] simulated the crack growth process of two types of specimens using XFEM implemented in Abaqus software, and the simulated crack shapes agree well with the experimental results (see Fig. 6).

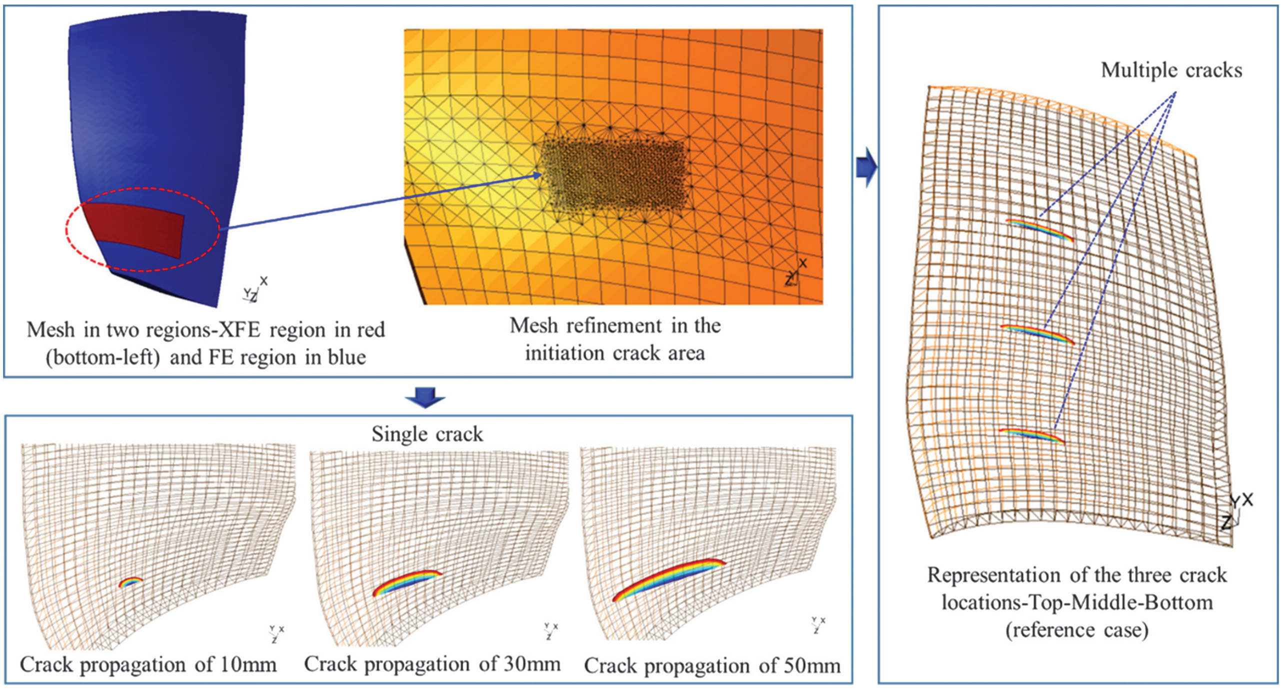


Fig. 5. Fatigue crack propagation on a rotor blade using XFEM [23].

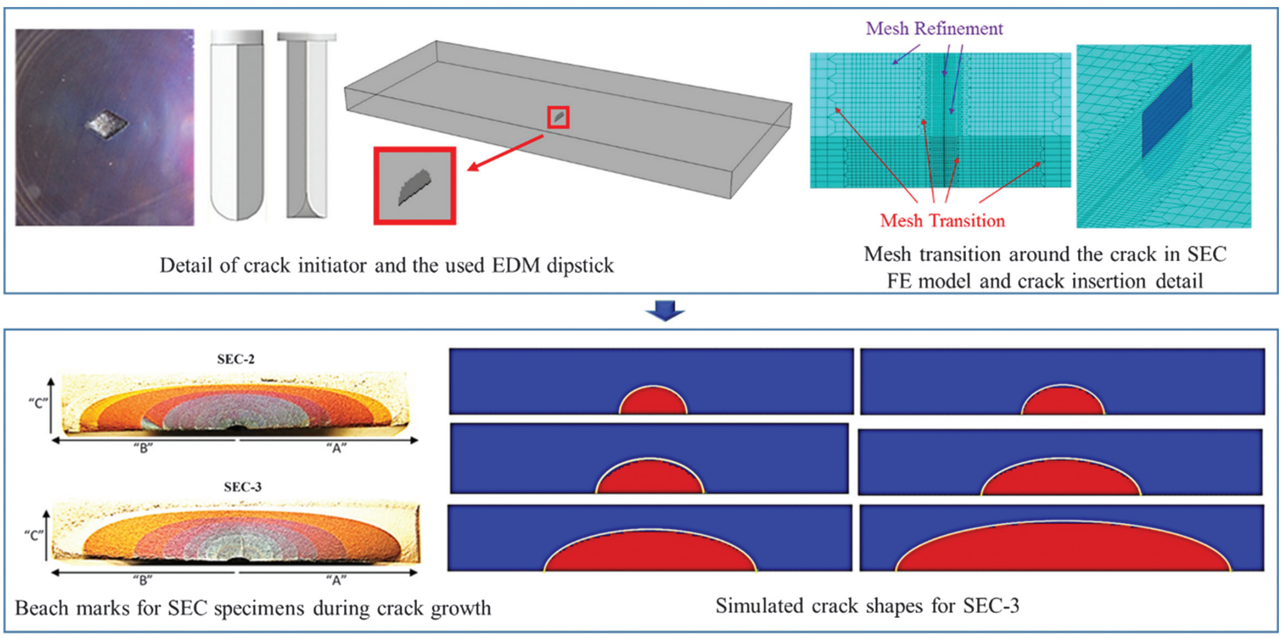


Fig. 6. Fatigue crack propagation paths of specimens using XFEM [25].

C. MESHLESS METHOD

The meshless method is based on the idea of using the nodes in the influence zone of a selected point to construct its approximation space [26]. This method does not need to generate mesh in numerical calculation but constructs discrete governing equations of interpolation functions according to some arbitrarily distributed coordinate points. It can be understood as constructing approximate functions according to nodes to eliminate problems caused by dependence on mesh.

The meshless methods are divided into two categories according to whether integration is required, namely collocation-type and Galerkin-type [27]. At present, the popular collocation-type meshless method mainly includes smoothed particle hydrodynamics, radial basis collocation method, finite point method, Hp-meshless cloud method, generalized DFM, moving particle semi-implicit method, subdomain radial basis collocation method, radial point interpolation collocation method, reproducing kernel collocation method, gradient smoothing collocation method, stabilized collocation method, etc. Galerkin-type meshless methods include element free Galerkin method, reproducing kernel particle method, generalized FE method, Hp-meshless cloud method, partition of unity method, meshless local Petrov–Galerkin method, method of finite spheres, radial point interpolation method, smoothed particle Galerkin method, etc. Yang *et al.* [26] proposed a multiscale method for crack growth where the coarse and refined areas are modeled by a differential reproducing kernel particle method and a molecular statics approach, respectively. Crack propagations are simulated using the phantom node method and breaking of bonds in the coarse and refined scale areas, respectively. Khosravifard *et al.* [28] compared the advantages and disadvantages of the element-free Galerkin method and the meshless radial point interpolation method for simulating crack growth problems.

D. BOUNDARY ELEMENT METHOD

BEM transforms the differential equations in the domain into boundary integral equations by using basic solutions. At the same time, BEM adopts the element discrete technology similar to the FEM. By discretizing the boundary into boundary elements, the boundary integral equations are discretized into algebraic equations and solved by numerical methods. The solution of the boundary integral equation of the original problem is obtained. The characteristic of the BEM is that the dimension of the original problem is reduced by one dimension (the two-dimensional problem is turned into a one-dimensional problem on the boundary line, and the three-dimensional problem is turned into a two-dimensional problem on the boundary surface). Citarella and Perrella [29] carried out a 3D crack propagation simulation of a complicated geometry specimen using Beasy software based on BEM considering mode coupling effects. The proposed method calculates the SIFs by the crack opening displacement method and determines the crack propagation direction using the minimum strain energy density criterion. Citarella *et al.* [30] performed a 3D crack growth simulation using Beasy software based on BEM and Zencrack and Cracktracer3D based on FEM. The J-integral method is used to calculate SIFs for Beasy and Zencrack and the quarter point element stress method for Cracktracer3D. The crack propagation directions are determined using the minimum strain energy density and maximum tangential stress criteria for Beasy, the maximum energy release rate and maximum tangential stress for Zencrack, and the maximum principal asymptotic stress for Cracktracer3D. Based on Beasy software using dual BEM, Ramezani *et al.* [31] analyzed SIFs of Modes, I, II, and III for semi-elliptical cracks in round bars under torsional loads.

E. SCALED BOUNDARY FEM

To combine the best features of the domain-based approaches such as FEM and BEM, the scaled boundary FEM (SBFEM) is developed [32,33], which reduces the spatial dimension using BEM and like FEM, and it does not need knowledge of a Green's function. The boundary of the domain is discretized using FEM, and it facilitates the computation of the angular variation of the unknown fields. These improvements can ensure the accuracy of the solution without local mesh refinement around the crack tip or other stress singularity points.

FEM-dual boundary element method (DBEM) can be regarded as one of SBFEM. It is a coupled method combining the advantages of FEM in performing complicated analyses with the benefits of DBEM in the process of tackling crack growth. Generally, an FEM is used to solve the global problem and calculate displacement and stress. The boundary conditions extracted from the FE model are applied to a DBM submodel. Citarella *et al.* [34] performed crack propagation simulation in aircraft engine vane by FEM-DBEM where a subdomain is firstly identified through a 3D FEM mesh in Ansys software for the uncracked structure and the crack initiation and growth are simulated by DBEM in Beasy software (see Fig. 7). Based on a similar method, crack propagation paths are also simulated for a low-pressure aeroengine turbine vane segment [35,36].

F. SUMMARY

Different crack propagation simulation methods are listed in Table I. The table shows the simulation ideas of crack growth, and the advantages and disadvantages of these methods. By summarizing the current literature research progress in blade crack propagation, it can be found that TFEM, XFEM, and FEM-DBEM (one of SBFEM) are the main methods used for complex structures, such as blades. The TFEM mainly uses commercial software Ansys or Abaqus combined with special crack propagation software such as Franc3D, Zencrack, and Cracktracer3D (mesh updating idea). XFEM has also been embedded in commercial software such as Ansys and Abaqus. FEM-DBEM mainly uses commercial software Ansys or Abaqus combined with special crack propagation software such as Beasy (the boundary element idea). Due to some shortcomings of the Meshless method and BEM, they need to combine with TFEM to solve more complex three-dimensional crack propagation problems.

III. MECHANICAL MODELS OF OPEN AND BREATHING CRACKS

There are two types of crack models available: open crack model and breathing crack model. The open crack is mainly

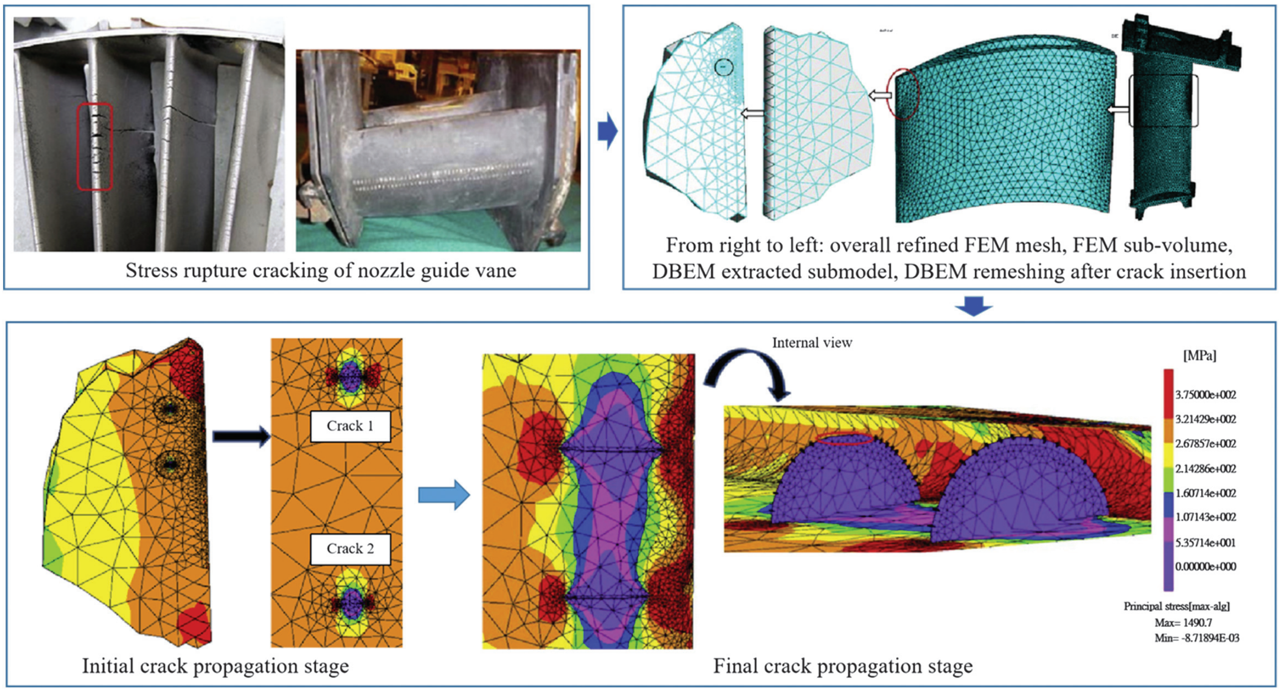


Fig. 7. Crack growth simulation of an aircraft engine vane using FEM-BEM [34].

Table I. Simulation methods for crack growth

References	Methods	Crack propagation simulation	Advantages	Disadvantages	Notes
Mangardich [11,12] Shlyannikov [14] Poursaeidi [15] Liu [16] Canale [17] Salehnasab [18]	TFEM	Commercial FE software (Ansys, Abaqus, etc.) combined with fracture mechanics tools (Franc3D, Zencrack, etc.)	A robust and industry-accepted numerical method	Remeshing after each propagation step, requiring additional post-processing techniques to extract the SIFs	Fracture mechanics software makes up for the shortcomings of the TFEM. At present, TFEM has been widely used in blade crack propagation simulation
Padovan [20] Ci [21]		Birth-death element	The theory and implementation process are relatively simple	The method can simulate 2D plane crack propagation accurately, but it is difficult to simulate three-dimensional crack propagation	In order to improve the accuracy, it is necessary to refine mesh in the crack area, and the element can deal with large deformation and strain
Hu [22] Dompierre [23] Holl [24] Bergara [25]	XFEM	Augmenting the TFEM approximation basis with a priori known functions that can capture the local behavior, adopting level set method (LSM) to describe interface dynamic changes	There is no need to generate a mesh matching to crack-induced geometry discontinuities, having high efficiency	Introducing numerical difficulties in crack growth simulations such as the need for special integration techniques in the enriched regions	The method has been implemented in commercial software such as Abaqus and Ansys
Yang [26] Wang [27] Khosravifard [28]	Meshless method	Adding a new pair of nodes to express the newly formed crack surfaces as the crack grows	No mesh required which greatly simplifies pre-processing process, easy to construct high order shape function and carry out adaptive analysis	Imposition of essential boundary conditions, inaccuracies in defining the crack geometry, low calculation efficiency	The method has been implemented in commercial software such as LS-DYNA and AuToDYN

(continued)

Table I. (continued)

References	Methods	Crack propagation simulation	Advantages	Disadvantages	Notes
Citarella [29,30] Ramezani [31]	BEM	The crack propagation is similar to TFEM. For example, the stress intensity factors by the crack opening displacement (COD) method and the crack propagation direction using the minimum strain energy density criterion	Reducing the computational complexity because only the boundary needs to be discretized, can easily deal with infinite field problem and fracture problem	Difficult to apply to inhomogeneous media, and its application scope is not as wide as that of FEM; for general nonlinear problems, it is difficult to obtain satisfactory results	The method has been implemented in commercial software such as Beasy software
Song [32] Jiang [33] Citarella [34,35] Giannella [36]	SBFEM	The crack propagation using BEM	Combining the advantages of FEM and BEM	The implementation process is more complex and requires more experience than a single method	FEM-DBEM using Ansys/Abaqus and Beasy software is successively used to crack propagation simulation of actual stator and rotor blades

simulated using equivalent reduced section (ERS), cracked continuous bar or beam (CCBB), and local flexibility from fracture mechanics (LFFM). The breathing crack is simulated by bilinear spring model, harmonic function, nonlinear contact element, in-house developed crack element, and stress-based breathing crack model.

A. OPEN CRACK MODEL

An open crack model assumes that the crack is always open during vibration. Besides, the presence of cracks can change localized flexibility in a structural member. To evaluate the effect of open crack on the natural frequencies and vibration modes of structure, three basic methods (i.e. ERS, LFFM, and CCBB) are developed to characterize the open crack model [2].

ERS is suitable for notch-like cracks, using localized moments or reduced sections to simulate the effect of a notch on structural flexibility. The fundamental idea of this method is to quantify the reduction in local flexibility caused by the crack by considering the nonlinear stress distribution features induced by narrow slots [37].

LFFM employs lightweight torsional springs to simulate the increase in local flexibility caused by cracks [38]. The key concept of this method is to use equivalent springs to quantify the relationship between external loads and the stress field at the crack tip, which is used to calculate SIFs.

CCBB is more suitable for vibration analysis, including various modes, different boundary conditions, and the solution of lateral-torsional coupled vibrations. This method employs a one-dimensional reduction approach, integrating stress, strain, and displacement and correcting stress fields caused by the crack using local empirical functions. It also uses displacement fields from continuous flexible systems to better approximate stress and displacement distributions [39].

B. BREATHING CRACK MODEL

Using open crack models that assume constant crack opening during vibration to simulate the dynamic behavior of structures with fatigue cracks is unreasonable [4]. Several studies have indicated that under alternating loads, cracks exhibit nonlinear behavior, with their interfaces alternately opening and closing. This phenomenon, known as the "breathing effect," significantly influences the vibration response of structures with fatigue cracks [40].

1) BILINEAR SPRING MODEL. Bilinear spring model is widely used to simulate the nonlinear stiffness in mechanical systems, such as nonlinear boundaries and nonlinear elastic supports. It has also been extended to fatigue-cracked structures to reproduce the breathing effect [41]. In the bilinear spring model, the breathing crack is assumed to be in either a fully open state or a fully closed state. Chu *et al.* [41] presented a closed-form solution for a low-frequency excitation of a bilinear oscillator using double-wave functions. They extended this approach to model a breathing crack with a bilinear forcing function, deriving the corresponding frequency spectrum and verifying it numerically. Chati *et al.* [42] modeled a cracked beam as a system with piecewise-linear stiffness to define the effective natural frequency of the cracked beam, utilizing the concept of a "bilinear frequency." The effective natural frequency can be defined as $\omega_0 = \frac{2\omega_1\omega_2}{\omega_1+\omega_2}$, where ω_1 and ω_2 denote the natural frequencies under open and closed crack states, respectively. Chondros *et al.* [43] considered a breathing crack in a one-dimensional continuous Euler–Bernoulli beam model with bilinear characteristics. By comparing experimental results with analytical results for aluminum beams with fatigue cracks, it was observed that, as the crack depth increases, the vibration frequency variations in the case of breathing crack were less pronounced compared to fully open crack. Peng *et al.* [44] established a cracked beam model with bilinear stiffness based on FEM and used nonlinear output frequency response functions (NOFRFs) to detect cracks. The results indicated that NOFRFs are highly sensitive indicators for cracked beams, with larger NOFRF values corresponding to larger crack sizes. To realize the breathing crack in a continuous Timoshenko beam model, Wei *et al.* [45] used the sign function to simulate the bilinear stiffness. The super- and sub-resonance peaks can be observed in the forced vibrations considering different exciting frequencies. For the twisted actual aeroengine blade with edging crack, Xiong *et al.* [46] established a FE model considering variable

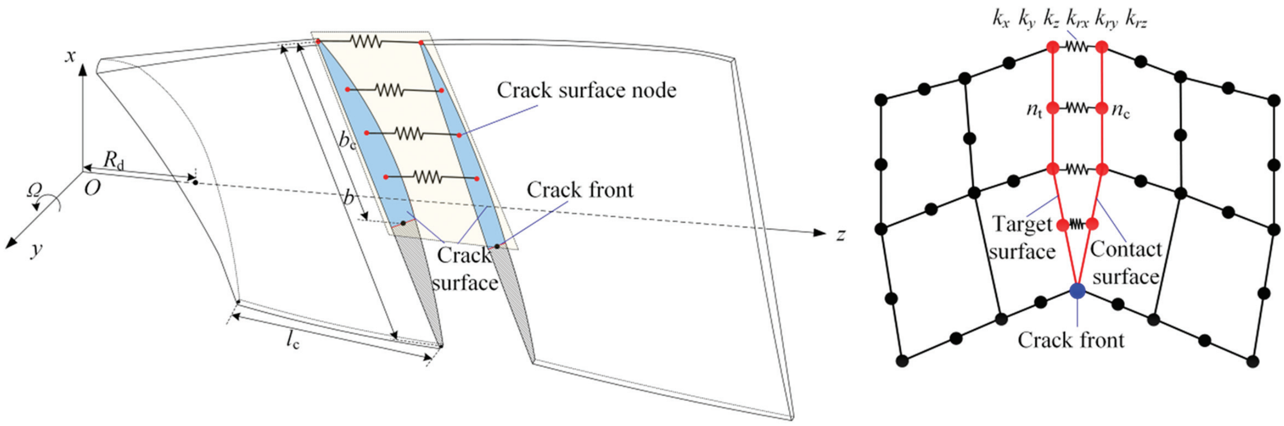


Fig. 8. FE model for a twisted actual aeroengine blade with breathing crack using linear spring [46].

cross-section using 3D shell element, as shown in Fig. 8. The crack surfaces are connected by springs, and the breathing effect is realized by the relative axial displacement between the nodes on the target surface (node n_t) and contact surface (node n_c).

2) HARMONIC FUNCTION MODEL. In reality, partial crack closure may result from interference, wedge entry, and elastic restraint of the plastic zone's wake. In other words, there may be a transition between the fully open state and fully closed state, a partially closed state [47,48]. As the load fluctuates, the stiffness of a structure containing a real fatigue crack may change. To simulate the continuous change of crack respiration, a harmonic function model is proposed. Abraham *et al.* [49] developed a dynamic model for breathing cracks using two cantilevered beam segments separated by the crack, which are connected by time-varying connection matrices expressed as Fourier series to reflect the interaction forces. Lagrange multipliers are employed to enforce continuity constraints when the crack is closed. Cheng *et al.* [40] simplified a cantilevered beam into a single-degree-of-freedom (SDOF) model and used cosine functions to simulate breathing cracks. In their model, the stiffness of the cracked beam is considered as:

$$k(t) = k_o + \frac{1}{2}(k_c - k_o)(1 + \cos \omega t) \tag{1}$$

where k_o and k_c are the stiffnesses of the open state and closed state, and ω is the fundamental frequency.

Based on Cheng's cosine function-based model [40], Xu *et al.* [50] simulated the cracked blade by a SDOF lumped mass model considering continuous opening and closing process of a breathing crack, as shown in Fig. 9. They emphasized that as the crack depth and the distance

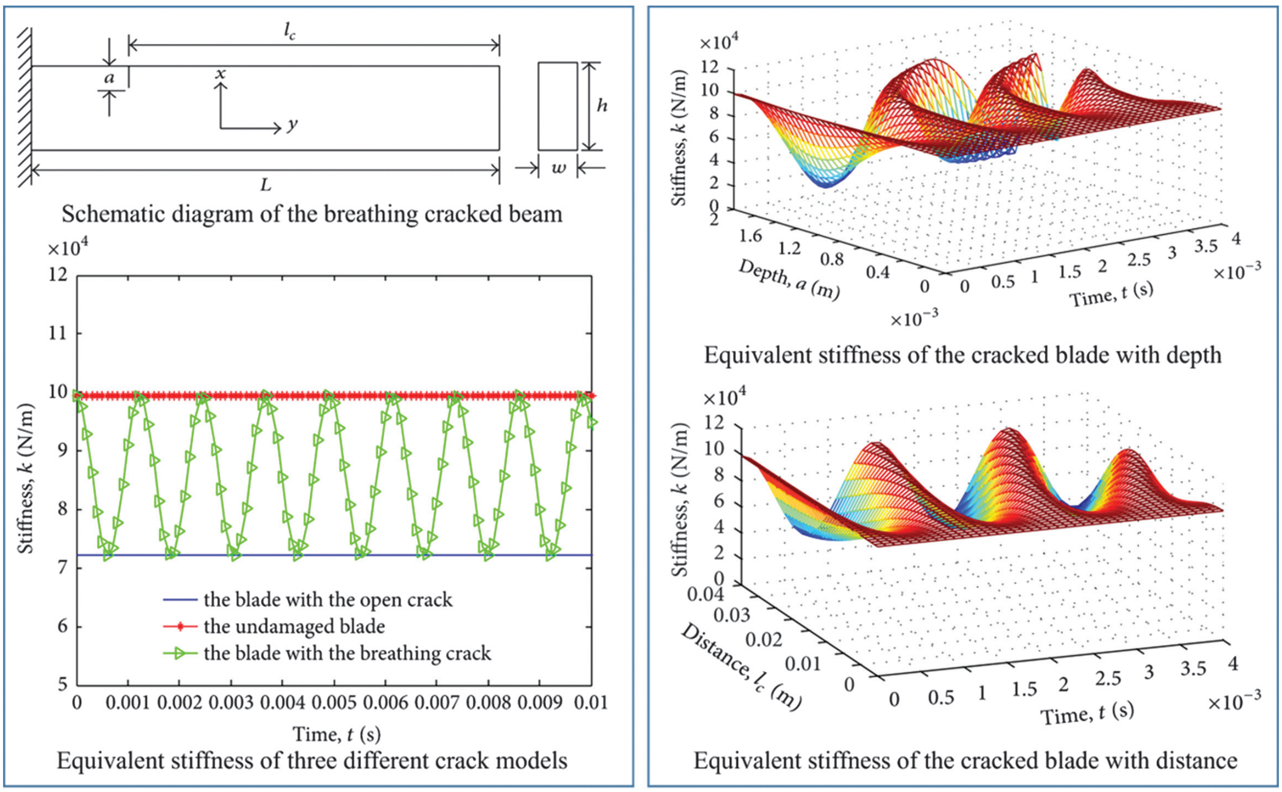


Fig. 9. Equivalent stiffness of the cracked blade [50].

between the crack and blade tip increase, the amplitude of the cosine function also increases, thereby intensifying the nonlinearity of the system.

Xu *et al.* [51] introduced a novel nonlinear pseudo-force (NPF) to elucidate the mechanism behind the generation of higher-order harmonics caused by breathing. In their model, the NPF was assumed to be related to cosine functions. Different from Cheng's model [40], Rezaee *et al.* [52] assumed that the local stiffness variation caused by breathing cracks follows a harmonic form related to the amplitude under fully closed and fully open conditions. The equivalent stiffness is determined as:

$$k(t) = k_o + \frac{k_c - k_o}{2} \times \left(1 + \cos\left(2\pi\left(1 - \frac{A_o^2 + A_c^2}{A_o A_c (A_o - A_c)} A + \frac{A_o + A_c}{A_o A_c (A_o - A_c)} A^2\right)\right)\right) \tag{2}$$

where A_o, A_c, and A are the amplitudes of a specified point corresponding to fully open, fully closed, and breathing state of the cracked beam.

3) NONLINEAR CONTACT MODEL. The FEM is widely employed for vibration analysis of complex systems. Many researchers have explored the nonlinear vibrations of cracks based on the nonlinear contact model. The contact pair elements are used to simulate the breathing effect resulting from the interaction of the crack surfaces [53]. Andreaus *et al.* [54,55] employed a 2D contact model to simulate the breathing behavior between crack surfaces and analyzed the bending vibration of cantilever-cracked beams. They investigated the influence of parameters such as crack location and depth, highlighting that the acceleration response at the tip node of the cantilever beam is sufficient to detect the existence of the crack and to identify crack parameters. Zeng *et al.* [53] investigated the dynamic characteristics of compressor blade cracks during the rotation process under different propagation levels. In their model, the blade crack is assumed to be a half-elliptical fatigue crack. To simulate the breathing behavior between cracked surfaces, they employed 3D 8-node surface-to-surface contact elements (Targe170 and Conta174 elements) in combination with the augmented Lagrangian contact algorithm, as depicted in Fig. 10. Tien *et al.* [56] proposed an efficient computational algorithm to calculate the transient and steady responses of the cracked beam with multiple contact pairs, which is referred to as the generalized hybrid symbolic-numeric computation. In addition to the aforementioned standard FE models, some researchers have investigated breathing cracks using spectral FEMs. Yu *et al.* [57] proposed a spectral gap element to simulate the crack for analyzing wave propagation in cracked structures and adopted separable hard contact pairs to realize the crack breathing effect. Saito *et al.* have made significant contributions to FE model reduction in structures with cracks [58]. They used the Craig–Bampton method of component mode synthesis (CMS) [59] and Guyan reduction [60] to generate a reduced-order FE model for the cracked structures and investigated the nonlinear vibration responses and nonlinear resonant frequencies of the cracked blade [61]. They also proposed an efficient approach combining a hybrid-interface CMS method, a contact detection algorithm, and an alternating frequency/time-domain method for the cracked-bladed disk system to analyze the effects of the blade crack on blade response and disk mistuning [62].

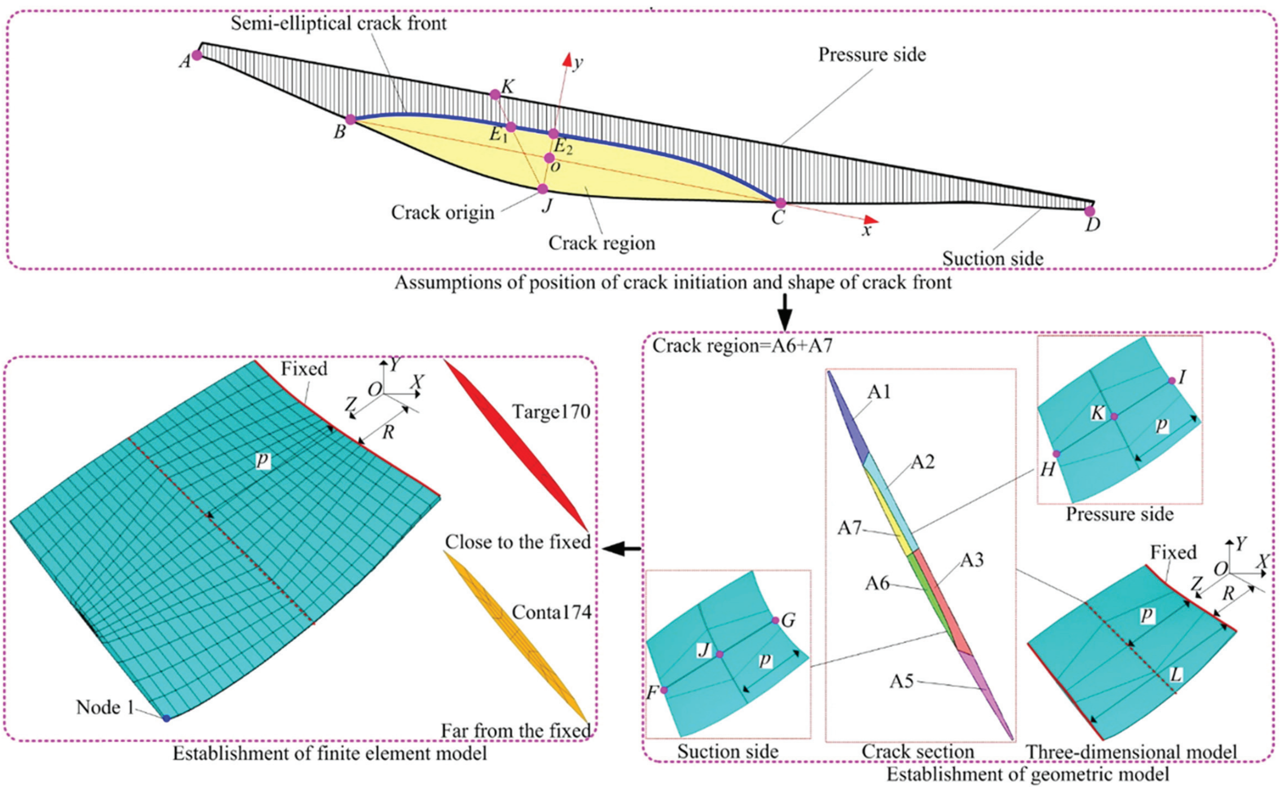


Fig. 10. FE model for cracked compressor blade with breathing crack using nonlinear contact model [53].

4) IN-HOUSE DEVELOPED CRACK ELEMENT. In order to improve efficiency, some developed crack elements considering the breathing effect without establishing the contact element are proposed. Kim *et al.* [63] proposed a crack element based on Castigliano's theorem, the cracked dependent stiffness is calculated through the fracture mechanics approach, and the breathing function is used to realize the stiffness changes during the vibration. It should be noted that the in-house developed crack element is different from the bilinear model. The bilinear model typically connects two elastic bodies or nodes using time-varying linear springs, with the time-varying stiffness implemented through a breathing function. In contrast, the in-house developed crack element adds a time-varying element stiffness to the healthy element stiffness, and this time-varying element stiffness is often achieved by multiplying a constant stiffness matrix with a breathing function. Based on Castigliano's theorem, Zhao *et al.* [64] introduced a novel model for a cracked beam element (CBE). This model accounts for the breathing behavior of cracks by considering variations in the crack contact region. The contact region is defined through six different contact types based on stress distribution during vibration. Liu *et al.* [65] developed a crack hexahedral element model (CHEM) based on the strain energy release rate method. The in-house developed crack element (hexahedral element) is first derived with strain energy release rate method, where correction of SIFs of crack front and formulation of load distribution of crack surface are carried out to improve the modeling accuracy. They introduced a breathing function to simulate the opening and closing effect induced by alternating loads. Furthermore, Liu *et al.* [66] analyzed nonlinear vibrations induced by crack breathing effect and alternating loads using a hexahedral element-based CHEM.

5) STRESS-BASED BREATHING CRACK MODEL. The combined effect of blade bending load and centrifugal force causes the breathing behavior of cracked blade. Xie *et al.* [67] recently presented a stress-based breathing crack model to simulate the rotating blade with breathing crack by judging the stress state at the crack region, thereby relating breathing cracks to centrifugal effects and loads. Based on Xie's model, Yang *et al.* [47,68] established an improved stress-based breathing crack model, in which the centrifugal stiffening, spinning softening, and Coriolis effect of the rotating blade are considered and the breathing function is modified based on elastic fracture mechanics. It is important to note that in Xie's and Yang's model, the centrifugal additional bending moment with a correction factor must be considered to account for the shifting of the vibration average caused by the crack. Wu *et al.* [69] established a cracked rotating blade based on the Timoshenko beam theory. The blade crack, taking into account axial-bending coupling, was developed based on the released strain energy. The presence of the breathing crack was determined by the combined effects of tensile stress and bending stress, as shown in Fig. 11, and no approximate functions or modified centrifugal additional moments were used.

C. SUMMARY

The available crack models are listed in Table II. The table shows the advantages and disadvantages of these models. By summarizing the current literature research progress in crack modeling, it can be found that ERS, LFFFM, and CCBOB are the main methods used for open crack modeling, wherein LFFFM and CCBOB have more potential advantages in simulating the opening crack behaviors than ERS. Due to the limitation of opening crack model in characterizing the breathing behavior of a crack, the breathing crack model can be divided into two categories: contact model (nonlinear contact model) and non-contact model (bilinear spring model, harmonic function model, in-house developed crack element, and stress-based breathing crack model). The breathing effect in the contact model is achieved through contact pair judgment, whereas, in the non-contact model, it is achieved through conditional judgment. Considering the crack breathing effect in non-contact models, the governing equation of the cracked system can be expressed as [47]:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + (\mathbf{K} - f_{\text{breathing}}\Delta\mathbf{K})\mathbf{q} = \mathbf{F} \tag{3}$$

where **M**, **C**, **K**, **q**, and **F** are the mass matrix, damping matrix, stiffness matrix, generalized displacement vector, and generalized force vector, respectively; $\Delta\mathbf{K}$ is a crack-dependent stiffness, which is usually calculated based on released strain energy; and $f_{\text{breathing}}$ is the breathing function for different crack non-contact models. The expression can be written as:

$$f_{\text{breathing}}(t) = \begin{cases} 1, & \text{Open crack model} \\ \frac{1-\text{sign}}{2}, & \text{Bilinear spring model} \\ \frac{1-\cos(\omega t)}{2}, & \text{Harmonic function model} \\ f(y_c), & \text{Stess-based breathing crack model} \end{cases} \tag{4}$$

where y_c represents the position information of the crack tip and determines the stiffness of the fully open crack.

IV. DYNAMIC MODELING METHODS OF ROTATING BLADES

The investigation of dynamic characteristics of cracked blades necessitates a precise model as a fundamental basis for research. Existing studies primarily concentrate on the development of models for rotating cracked blades, employing lumped mass models, semi-analytical models, and FE models.

A. LUMPED MASS METHOD

LMM simplifies the healthy blade into multiple mass blocks, interconnected by springs. The crack can be considered as a local reduction in stiffness. LMM is mainly used for qualitative analysis of crack vibration characteristics. Douka *et al.* [70] developed a SDOF lumped-parameter model for cracked beams, incorporating two stiffness values—one large for the closed state and one small for the open state. They employed periodic functions to simulate the time-varying stiffness caused by practice breathing cracks. To analyze both simulated and experimental results, they utilized empirical mode decomposition and the Hilbert transform. Yang *et al.* [71] established a simplified model for a blade with tenon and shroud based on lumped-parameter model in which each blade is simplified as a 2-DOF model, the blade crack is simulated by the additional flexibility stiffness, and the crack breathing effect is realized by the cosine function. Neves *et al.* [72] adopted the discrete

Table II. Mechanical models of opening and breathing crack

Crack type	References	Methods	Advantages	Disadvantages	Notes
Opening crack	Zheng [37]	Equivalent reduced section	Simple modeling and suitable for analytical analysis	Not suitable for real crack modeling	Localized moments or reduced section to modify the structural flexibility caused by the notch-like crack
	Chondros [38]	Local flexibility from fracture mechanics	Suitable for a cracked beam under arbitrary load conditions	Simple crack shape and limitations of crack tip hypothesis	Massless torsional spring to simulate the crack-induced local flexibility increase
	Chondros [39]	Cracked continuous bar or beam	Suitable for the vibration analysis of extended modes, varying boundaries, and the solution of lateral-torsional coupling vibration	Limited to vibration analysis of one-dimensional beam-like structure	The method has great potential for development in analyzing the vibration of a cracked structure
Breathing crack	Chu [41] Chati [42] Chondros [43] Peng [44] Wei [45] Xiong [46]	Bilinear spring	A simple principle and easy to implement in various models	The transition state between crack opening and closing cannot be considered	It is assumed that the crack is in two extreme states of full opening or full closing (two states)
	Abraham [49] Cheng [40] Xu [50] Xu [51] Rezaee [52]	Harmonic function model	The transitional states of both fully open and fully closed crack can be considered	It is based on idealized assumptions, relying on harmonic functions, which may not always accurately represent real-world conditions	It is the smoothing trigonometric function of the vibration displacement (three states)
	Andreaus [54][55] Zeng [53] Tien [56] Yu [57] Saito [58–62]	Nonlinear contact model	It offers high computational accuracy and is suitable for various complex crack shapes, providing rich response information through the contact area	It has lower computational efficiency and requires combination with dimensionality reduction methods and alternating frequency/time methods to improve efficiency	The breathing effect between crack surfaces is simulated by contact element (three states)
	Kim [63] Zhao [64] Liu [65,66]	In-house developed crack element	For regular shapes, it has high computational accuracy while being more computationally efficient than nonlinear contact models	It is suitable for simulating regular crack patterns but may not handle complex crack simulations effectively	A crack element is developed, which can consider the effect of breathing crack (three states)
	Xie [67] Yang [47,68] Wu [69]	Stress-based breathing crack model	It is based on the tensile and bending stresses, thus having physical interpretability	It is primarily based on beam theory and not be suitable for modeling complex blades and cracks	The model considers the combined effects of the centrifugal stress and the bending stress (three states)

element method in conjunction with an existing result from fracture mechanics that takes into account the local flexibility of a cracked beam. The dynamic behavior of a cantilever beam and a beam free of support conditions is investigated, and the effect of the presence of a crack is analyzed. Hou [73] constructed a theoretical model for the bladed disk based on LMM, as shown in Fig. 12. They represented cracks at the blade roots using the flexibility matrix method. By analyzing the phenomena of modal localization and natural frequency reduction under various coupling ratios between blades and disks, they explained the mechanism of bladed disk mistuning induced by blade cracks. Xu *et al.* [74] established blade cracks based on a local flexibility model and a cracked blade-rigid disk model using a lumped-parameter approach. They introduced local parameter indices to quantitatively analyze the effect of crack location and crack depth on modal localization in the blades.

B. SEMI-ANALYTICAL METHOD

SAM is primarily established based on beam theory and plate-shell theory to develop dynamic models for cracked blades. These models balance computational accuracy and efficiency. Wu *et al.* [75] developed a semi-analytical model for rotating cracked blades. They obtained the local flexibility of the crack based on strain energy release rate, derived discrete motion equations using energy and

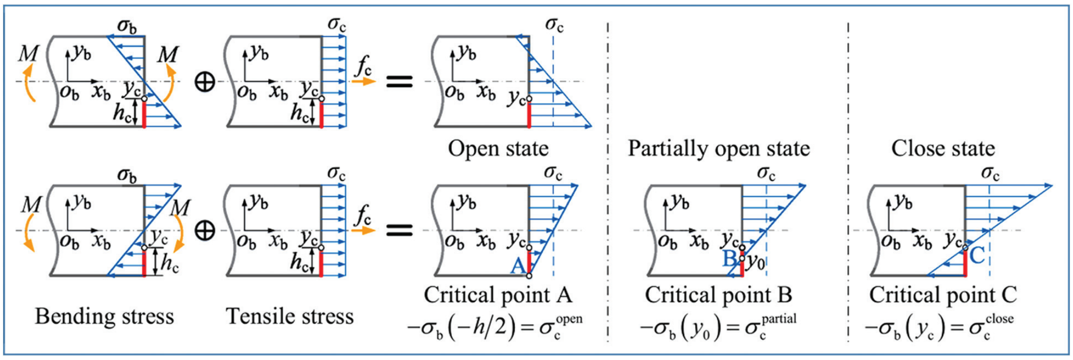


Fig. 11. Schematic diagram of bending stress and tensile stress at different states [69].

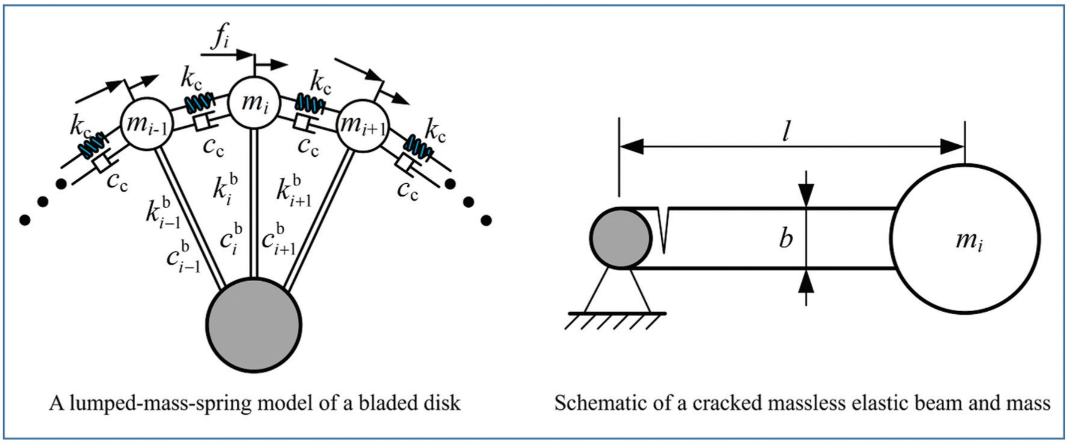


Fig. 12. Schematic of a theoretical model for bladed disk based on lumped mass method [73].

weighted residual methods, and analyzed the influence of crack depth and location on natural frequencies at different rotational speeds. Al-Said *et al.* [76] focused their attention on a rotating fractured beam that is divided into two segments. To simulate the crack, a massless torsion spring is used to connect the two segments. Using the Lagrangian equations and the assumed mode method, the differential equations of motion are derived for the rotating cracked beam. The investigation examines the effects of crack depth, shear deformation, and rotational speed on the beam's natural characteristics. A rotating Euler–Bernoulli cracked beam model was developed by Yashar *et al.* [77], in which the crack was equivalently represented by a massless spring. Using stress intensity parameters, they computed the increased strain energy caused by the crack. Lastly, they used commercial software to check the results of solving for eigenvalues using the Rayleigh–Ritz method. Their model, however, did not account for the consequences of crack breathing. Wu *et al.* [69] established a semi-analytical model based on Hamilton principle and assumed mode method. In their model, the rotational effects such as centrifugal stiffness, spin softness, and Coriolis effects are considered. Zhang *et al.* [78] derived a three-dimensional blade tip clearance dynamic model for rotating cracked blades based on the Timoshenko beam theory, as shown in Fig. 13. By considering the coupling effects

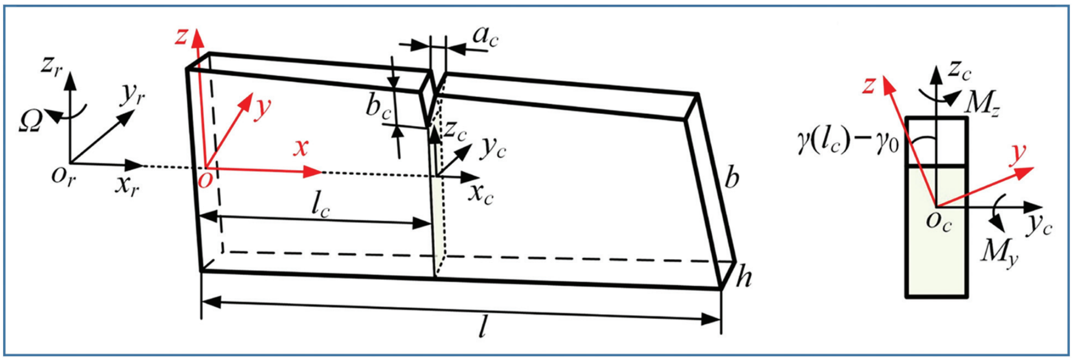
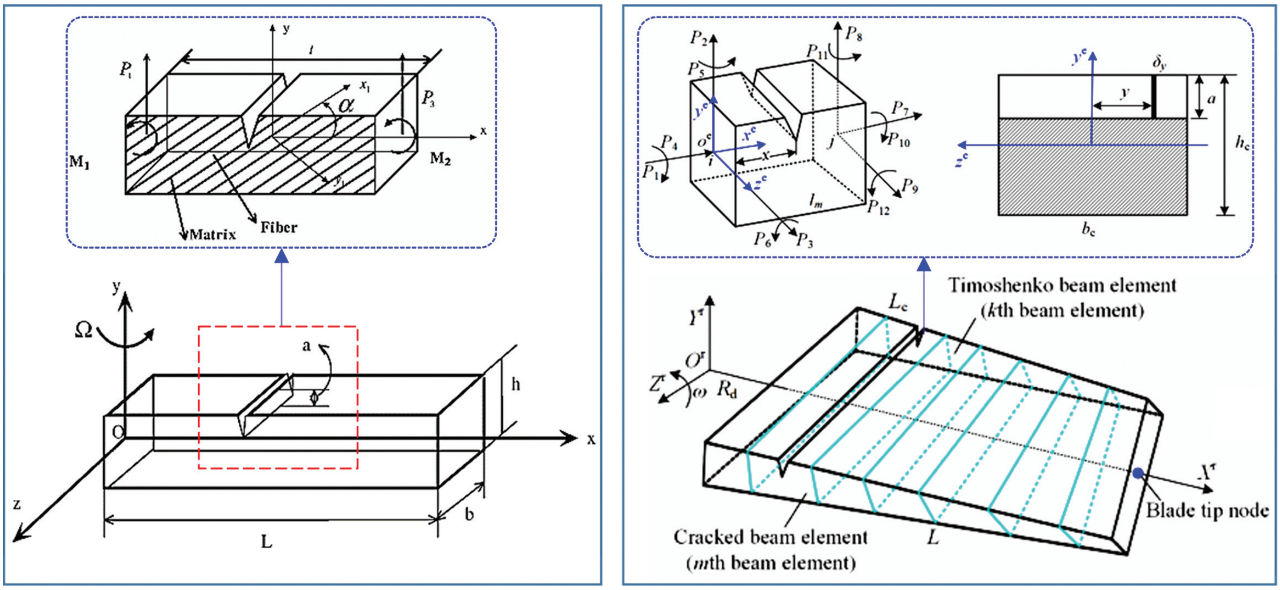


Fig.13. Schematic of cracked blade model with trailing edge crack based on semi-analytical method [78].



In-plane bending of composite cracked beam

Bending-axial-torsion of double tapered cracked beam

Fig. 14. Configuration of rotating cantilever beam with a breathing crack [64].

of centrifugal stress, axial tensile stress, and circumferential bending stress, they realized the crack breathing effect in twisted blades.

C. FINITE ELEMENT METHOD

1) FEM USING BEAM ELEMENT. Considering the dominant bending vibration of a blade with a relatively high aspect, many scholars usually simplify the blade as a beam model to study corresponding dynamic characteristics due to its maturational theory and simple modeling. Taking the unidirectional reinforced polyimide within a beam into account, Kim and Kim [63] built the in-plane bending FE model of a rotating cantilever beam with a uniformly penetrating crack. In the model, only the centrifugal stiffening effect was included, and the breathing behavior of a CBE was characterized via the stiffness matrix switching back and forth between fully open and fully closed states. Via the Timoshenko beam theory, Zhao *et al.* [64] developed the FE model of a double-tapered and pre-twisted cantilever beam with a uniformly penetrating crack under the combined action of centrifugal and aerodynamic force. In the model, the rotation-induced centrifugal stiffening, spin softening, and Coriolis effects were all included. Moreover, the breathing behaviors of a crack among fully open, partially closed, and fully closed states were also perfectly considered via the modification of contact stress status between crack surfaces (see Fig. 14).

2) FEM USING SHELL ELEMENT. The shell FE model has high precision and efficiency and can simulate penetrated cracks. Sun *et al.* [79] proposed a shell FE model with variable-section and twisted-shape features for rotating blades, which has good accuracy compared to the analytical or semi-analytical models, and higher computer efficiency than the solid FE models. However, their model does not incorporate any crack faults. Xiong *et al.* [46] established a breathing model of the rotating blade with an edging crack by the linear spring. By judging the normal distance of the node corresponding to the contact surface, if the normal distance is less than zero, the spring is applied. It should be noted that the FE model of the cracked blade using the shell element is not suitable for simulating non-penetrated cracks.

3) FEM USING SOLID ELEMENT. The solid FE model has high precision and can simulate complicated crack types. Zeng *et al.* [53] developed an FE model for variable cross-section twisted compressor blades. They considered the rotational effects of the blades and incorporated a relatively accurate crack propagation path (semi-elliptical cracks). Using contact elements, they investigated the influence of angular acceleration, excitation amplitude, and crack parameters on the nonlinear vibrations induced by breathing cracks during the acceleration process. However, the calculation efficiency is low, so it is necessary to combine the dimension reduction method to improve the efficiency. Saito *et al.* [59] employed the Craig–Bampton modal synthesis method, known as CMS, to establish a reduced FE model that retained only the nodes in the crack plane. They utilized piecewise linear terms to simulate the breathing phenomenon and solved the model using a mixed time/frequency domain technique involving multiple harmonics. Similarly, Jung *et al.* [80], and D'Souza *et al.* [81], using Ansys software, retained crack surface nodes and employed a modal synthesis method to establish reduced-order models of cracked blades on the rotor. They utilized a time-domain and frequency-domain alternation approach to solve the nonlinear equations of steady-state motion.

4) FEM USING HYBRID ELEMENT. Low-dimensional elements such as the beam element and lumped mass-spring element have high efficiency but low precision in characterizing the physical model of a rotating cracked blade, while high-dimensional elements such as plane and solid elements have low efficiency but high precision in representing the physical model of a rotating cracked blade. Therefore, a natural thought is whether to integrate the low-dimensional elements with high-dimensional elements to model the cracked rotating blade. Some researchers have

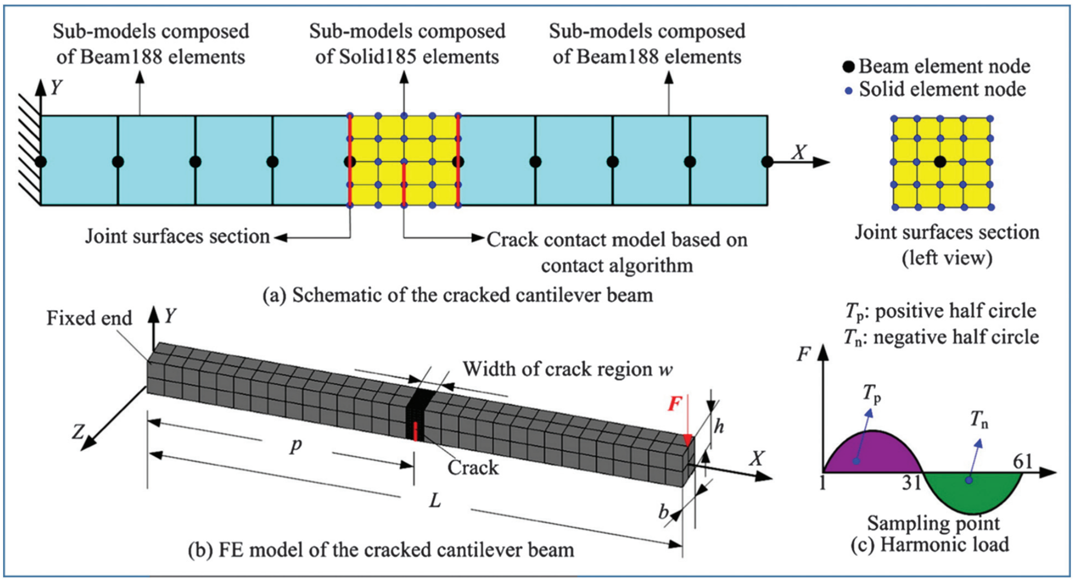


Fig. 15. Solid-beam hybrid finite elements model for breathing cracked beam [83].

utilized this thought to make some achievements. In the existing cracked blade models employing mixed elements, the structure is usually divided into regions with and without cracks. The non-cracked regions are simulated using beam elements, while the cracked regions are modeled using plane or solid elements. This approach significantly reduces the DOFs in the model, thereby enhancing computational efficiency while maintaining response accuracy. Using Ansys software, Ma *et al.* [82] considered impact and sinusoidal excitations. They constructed a straight crack beam model using Beam188 beam elements and 8-node Plane183 elements. Contact elements were employed to simulate the breathing effect of the crack. Similarly, they extended this approach to create a mixed-element model for oblique crack beams, with the crack tip represented using 2D singular elements. For other types of cracks, Zeng *et al.* [83] used solid elements to simulate the cracked regions and beam elements for the crack-free regions, as shown in Fig. 15. They developed models for cantilever beams containing non-penetrating parabolic cracks, penetrating trapezoidal cracks, and uniformly penetrating cracks. The validity of these models was verified through comparisons with existing research findings. In the case of cantilever beams, rigid supports are not always applicable. Elastic supports and eccentric boundaries may exist in certain situations. Addressing this issue, Zhang and Ma [84] employed a mixed-element modeling approach. They used different contact elements to describe the breathing effect of the crack and offset boundaries, while spring elements were utilized to simulate elastic supports. This approach allowed them to construct FE models for cantilever beams with cracks and offset boundaries.

D. SUMMARY

Different modeling methods for rotating blades are listed in Table III. LMM has higher computational efficiency and is commonly used for understanding mechanisms, but it cannot fully simulate the complex geometric configurations of blades. SAM is widely used due to its high accuracy and high computational efficiency, but it is also difficult to model complex blade structures. The FE model has advantages in handling complex blade geometries, load conditions, material characteristics, and boundary conditions. However, FE model may suffer from computational inefficiency, prompting some researchers to employ techniques like model reduction and hybrid element modeling to enhance computational efficiency.

V. DYNAMIC CHARACTERISTICS OF ROTATING BLADES

A. NATURAL CHARACTERISTICS

As the crack weakens the structural stiffness of the blade, it will lead to changes in the natural frequencies and mode shapes. With the increase of the crack level, the natural frequency of different orders will show a downward trend, and frequency veerings and crossings may occur. Saito *et al.* [61] analyzed the mode interchanging and coupling phenomena under open crack and breathing crack conditions. It should be noted that only natural frequencies of the linear system were analyzed for the open crack condition and the nonlinear resonant frequencies were obtained using hybrid time-frequency domain method for the breathing crack condition. Based on simulation and experiment, Xiong *et al.* [85] investigated the actual crack propogation path and the change rules of resonance frequencies. The findings revealed that the resonance frequency exhibites a gradual decrease in the initial stage of crack growth, followes by a consistent decrease during the intermediate stage of crack growth, and ultimately a rapid decline in the severe crack growth stage. Xu *et al.* [86] diagnosed the rotor blade crack by the change of the blade natural frequency obtained from the measured signal using blade tip timing (BTT) and strain gauge, as shown in Fig. 16. It should be noted that BTT signals are reconstructed by a sparse reconstruction method named block-accelerated orthogonal least squares.

In addition to the analysis of the natural characteristic changes caused by cracks, many researchers have

Table III. Modeling methods of rotating blades

References	Methods	Advantages	Disadvantages
Douka [70] Yang [71] Neves [72] Hou [73] Xu [74]	Lumped mass method	Simple principle, high computational efficiency	Difficult to accurately determine the mechanical parameters, only low-order modes can be simulated, and the modeling accuracy is low
Wu [75] Al-Said [76] Yashar [77] Wu [69] Zhang [78]	Semi-analytical method	Offering a balance between precision and computational speed	Difficult to simulate the actual blade structure, such as the varying-thickness-twisted blade
Kim [63] Zhao [64]	Finite element method	Beam element: simple and efficient	Beam element: difficult to consider the torsional vibration of complicated blades
Xiong [46]		Shell element: high accuracy and efficiency, can simulate varying-thickness-twisted blade	Shell element: simplified blade shape and crack model
Zeng [53] Saito [59] Jung [80] D'Souza [81]		Solid element: high accuracy, can simulate complicated blade structures	Solid element: low efficiency
Ma [82] Zeng [83] Zhang [84]		Hybrid element: the crack region adopts an advanced element (shell, solid elements) and the non-crack area adopts a simple element (beam element)	Hybrid element: difficult to apply to actual blade structures

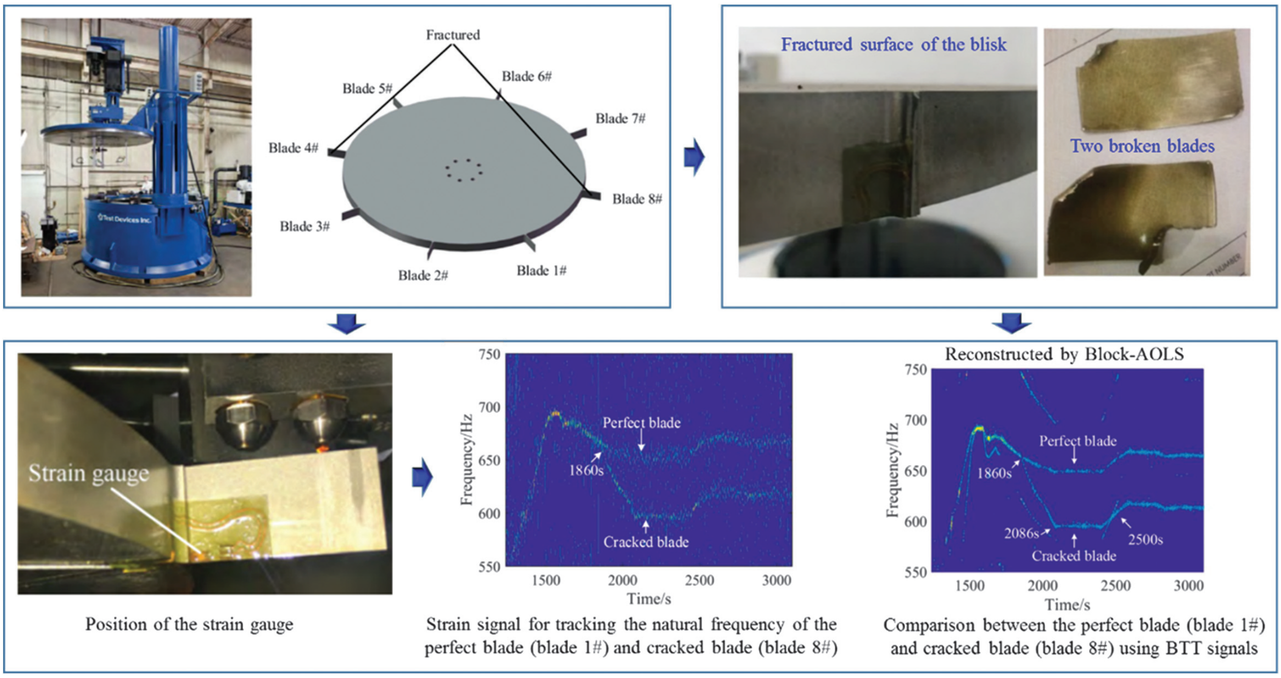


Fig. 16. Blade crack diagnosis by the change of natural frequency [86].

also estimated the location and size of cracks by the change of natural frequencies and mode shapes due to the crack affecting local flexibility. Zhang *et al.* [87] proposed a method for quantitative crack identification of a cantilever beam system and a simply supported beam system using multivariable wavelet FEM and particle swarm optimization. Zhang and Yan [88] identified multiple crack of a variable-cross-section cantilever beam structure by measuring the change of natural frequencies using Hilbert–Huang transform. Xiang and Liang [89] proposed a hybrid two-step method to detect the crack location and depth in beams where the crack location is firstly identified by the modal shape using the wavelet transform and then the crack depth is determined by a database established by wavelet FEM.

B. VIBRATION RESPONSE CHARACTERISTICS

Crack can lead to complicated dynamic behaviors and vibration features, such as superharmonic and subharmonic resonance responses, zero-frequency component (or constant component) due to asymmetric stiffness, phase portrait distortion phenomenon under different vibration directions, vibration behavior change due to crack coupled other nonlinearity, crack-face contact state changes, damping variation due to crack breathing, strong vibration localization caused by crack for blisk structures, etc.

1) SUPERHARMONIC AND SUBHARMONIC RESONANCE RESPONSES. Superharmonic and subharmonic resonance responses are easily excited under relatively small crack conditions, and these features are adopted to estimate the crack location and depths. Ma *et al.* [82] analyzed the dynamic characteristics of a slant-cracked cantilever beam, which is simulated using a beam-plane hybrid FE model and the crack is modeled by contact element. Their results show that gravity has a great influence on the nonlinear vibration response; under high frequency excitations, superharmonic and subharmonic resonance can be observed. In addition, the appearance of $3f_e$ (f_e denotes the excitation force frequency) and $0.5f_e$ can be regarded as distinguishable features for severe cracks. Aiming at a cantilever beam structure with a small crack (crack depth less than 20% of the beam section height), Giannini *et al.* [90] established the relation between the superharmonic or subharmonic component amplitude and damage characteristics and adopted harmonic damage surfaces to describe the dependency between the indicators and the crack location and depth.

The traditional frequency domain method mainly analyzes the components of multiple harmonics at superharmonic resonance state or fractional harmonics at subharmonic resonance state for crack diagnosis. The traditional method still has some difficulties in the diagnosis of weak cracks. In order to solve this deficiency, many researchers have proposed some indicators to improve its accuracy. Xiong *et al.* [46] proposed a novel crack fault diagnosis indicator for the rotational cracked blade based on the vibration energy method. Xu *et al.* [50] adopted vibration power flow analysis (VPFA) to obtain the nonlinear vibration features of rotating blades under small crack conditions, which show VPFA is more sensitive to a small crack than traditional displacement analysis. Peng *et al.* [44] proposed a novel concept of NOFRFs to identify the beam crack utilizing frequency domain information. The results indicate that the magnitudes of the calculated NOFRFs can serve as indicators of the crack levels, with larger values corresponding to more severe cracks.

2) ZERO-FREQUENCY COMPONENT (OR CONSTANT COMPONENT). The magnitudes of constant component increase with the increase of crack severity due to the crack-induced structural stiffness asymmetry, which can be viewed as a typical feature of quantifying crack level. Zeng *et al.* [83] studied the nonlinear vibration responses aiming at three types of crack, i.e., the non-penetrating parabolic crack, penetrating trapezoid crack, and uniform-penetrating crack. Their results show that the average vibration response of the cantilever beam with the single-edge down crack increases with the increasing crack level, i.e., the magnitude of the constant component will increase. Wu *et al.* [69] analyzed the axial-bending coupled vibration response of a rotating blade with breathing crack. They pointed out that the breathing crack will excite the fundamental frequency and its multiple frequency components, and there are constant components in the axial and bending displacement spectrum. Xie *et al.* [67] revealed the coupled influences of the centrifugal forces and the breathing crack on the nonlinear vibration responses of a rotating uniform-section blade. They found that the additional bending moment induced by centrifugal effects can intensify the asymmetry of the time-domain waveform with the increasing rotational speed, which can lead to the increase of the constant component. Yang *et al.* [47,91] further explained the cause of the constant component due to the additional bending moment caused by the coupling effects of crack and centrifugal force. Wu *et al.* [92] analyzed the effects of gravity load, rotor imbalance force, and aerodynamic load on blade tip vibration characteristics in the shaft-disk-cracked blade coupling system and investigated the effects of different dimensionless crack depths and crack positions on blade tip vibration characteristics. Their research suggests that gravity load can induce vibration of blades, and rotor imbalance force can cause static deformation of blades. The ratio of the constant component to rotational frequency amplitude and the ratio of the constant component to aerodynamic excitation frequency amplitude serve as effective metrics for assessing breathing crack.

3) PHASE PORTRAIT DISTORTION PHENOMENON. Phase portrait distortion phenomena can be observed due to the sharp changes of the relative velocity between the crack surfaces during closure, which is related to the repeated impact between crack interfaces [54]. The acceleration-velocity phase trajectories in the excitation direction and the lateral velocity-displacement phase trajectories perpendicular to the excitation direction can be used to identify the crack severity. If both phase trajectories are distorted, the crack may be more serious, and if only the velocity-displacement phase trajectories are distorted, it may indicate that only small cracks exist [83]. The phase portrait in axial direction is more sensitive than that in bending direction, for example, the larger offset in the center of the axial phase portrait is obvious compared with that in bending direction under the same crack severity [69]. Huang *et al.* [93] analyzed the geometric features of the phase portrait for a cantilever beam with breathing cracks and established some indicators based on these geometric features to identify the crack location and size. The results show that these indicators are more sensitive under superharmonic resonance conditions.

4) CHANGE OF CONTACT STATE OF CRACK SURFACE. In order to better reveal the nonlinear vibration mechanism induced by cracks, it is essential not only to scrutinize the alterations in natural characteristics and nonlinear vibration response induced by cracks but also to analyze the time-varying contact state of the crack surface resulting from the crack breathing process. By analyzing the contact state change of different crack surface positions using the in-house developed crack element and the contact element, Liu *et al.* [65] analyzed the change law of the open crack, closed crack, and transition state in the vibration process of the blade. Aiming at a pre-twisted cracked rotating blade, the change law of the breathing crack-induced time-varying stiffness is analyzed based on in-house breathing crack model [64]. The mapping relationship between crack, mechanical parameter change, and nonlinear vibration response can be better established by analyzing the change rule of stiffness. Wang *et al.* [94] systematically analyzed the changes in the contact behavior of cracked surfaces under varying depths, locations, and aerodynamic loads and introduced a quantitative index for breathing crack nonlinearity to offer a comprehensive assessment of the degree of nonlinearity. Based on the contact element in Ansys software, Zeng *et al.* [83] analyzed the contact pressure distribution for three types of crack. Their studies show that the local contact phenomenon can be observed under some vibration stages and the duration of open crack is longer than that of closed crack under large crack. Based on the similar method, Zeng *et al.* [53] quantified the change law of the crack surface contact state of a real cracked blade under the combined action of aerodynamic load and centrifugal load, and their research pointed out that the aerodynamic load played a leading role in crack breathing process at low speed conditions, and the centrifugal force played a leading role in the open crack at high speed conditions.

5) VIBRATION BEHAVIOR OF CRACK COUPLED OTHER NONLINEARITY. In addition to the nonlinearity caused by crack breathing, there may be other types of nonlinearity (geometry [95,96], boundary [84,97], and other fault-induced nonlinearity [98,99]) in the actual process of monitoring or diagnosis of cracked blades. The superposition of multiple nonlinearities may make the crack-induced fault mechanism more complicated and the difficulty of crack fault diagnosis will be greatly increased. Nandi and Neogy [97] analyzed the nonlinear vibration responses of cantilever beams with rigid offset support (the first type of nonlinearity) or with a breathing edge crack (the second type of nonlinearity) and observed the similar frequency features caused by two types of nonlinearities. Zhang *et al.* [84] analyzed the vibration responses of an elastic-support cantilever beam with crack and offset boundary using the contact element to simulate the nonlinear effect. It is found that vibration responses with offset boundary are similar to those under single-edge cracks, the nonlinearity is weakened due to the crack symmetry for double-edge symmetric crack, and the combined effects of offset boundary and down or up crack can weaken or strengthen the vibration. Tang *et al.* [98,99] analyzed blade-casing rubbing induced nonlinear vibration behaviors for the single blade and blisk with breathing crack, and the coupling vibration caused by two types of nonlinearities was deeply studied.

6) DAMPING VARIATION LAW. Generally speaking, cracks will lead to an increase in damping, and there are many explanations for the energy dissipation mechanism caused by cracks, such as the friction between crack interfaces and yield phenomenon near a crack tip [100]. The key to damage detection by damping is the initial level of damping, slenderness ratio of actual structure, and crack types (rectangular, circular cross-sections, etc.) [100]. Kharazan [101] performed some experiments to evaluate the effects of breathing cracks on damping characteristics for a cracked beam structure. The experimental results show that the damping coefficient increases with the increase of the crack depth and excitation amplitude. Considering the effects of nonlinear geometric stiffness and damping changes due to breathing crack, Kharazan [95] analyzed the nonlinear vibration behaviors (jump phenomenon, softening or hardening behavior) of a cantilever beam with a breathing crack. Considering the friction effect of the contact surface of the crack, the dynamic model of the cracked beam is established by using the contact element. With the increase of crack severity (large crack depth, small crack location), the damping ratio of the system shows an increasing trend [102].

7) STRONG VIBRATION LOCALIZATION CAUSED BY CRACK FOR BLISK STRUCTURES. Both mistuning and cracks can lead to strong localization of the forced response. For certain mode families, the cracked-blade-dominated response may appear at a significantly lower frequency and may also include unique characteristics such as a double resonance peak, as shown in Fig. 17 [62]. Huang and Kuang [103] studied the influences of a blade root crack

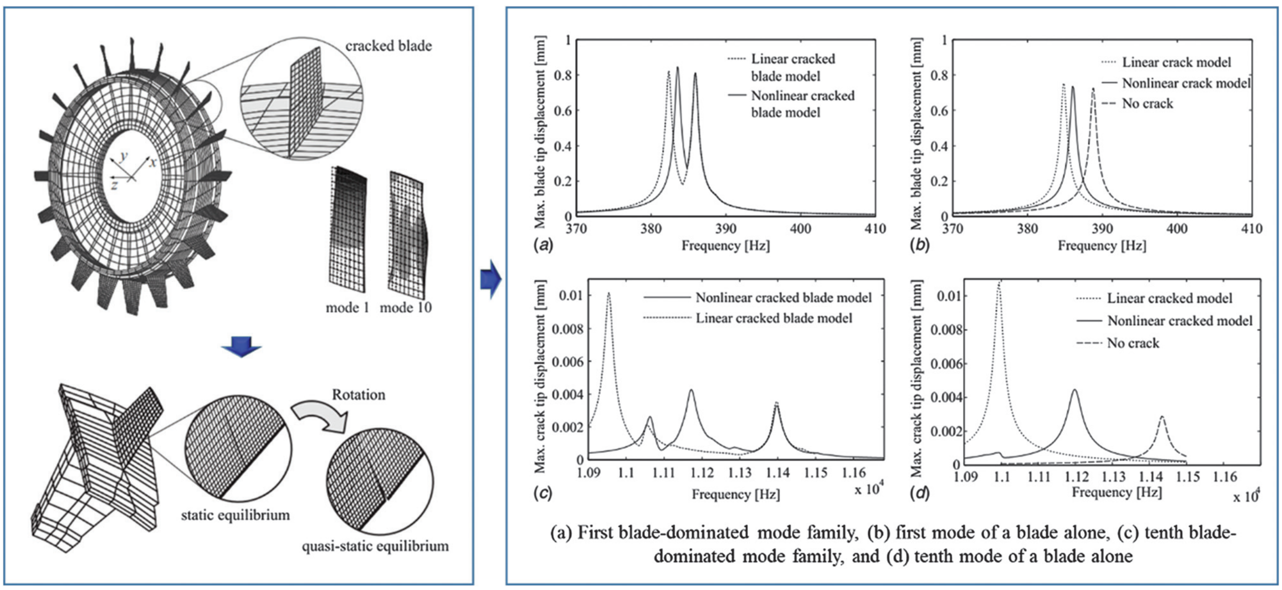


Fig. 17. Amplitude-frequency response of the bladed disk with a cracked blade and the blade alone [62].

on the stability of a blisk. Their simulation results show that the local crack may not only limit the disturbed energy in the blades to those near the cracked blade but also may change the instability region of the disordered blisk significantly. Marinescu *et al.* [104] proposed a new reduced-order modeling method, which can speed up calculations by several orders of magnitude, and analyzed the combined effects of mistuning and open crack on eigenvalue deviations. Xu *et al.* [105] presented a new method to identify crack depth and location for a hard-coated disk-blade structure based on mistuning idea. Wu *et al.* [106,107,108] established semi-analytical models for the coupling systems of bladed-disk and flexible dual rotor with blade crack and investigated the impact of cracks on the modal characteristics of the coupling systems. According to their research, the localized mode of the disk, induced by the blade crack, undergoes a transformation from the 2-nodal diameter mode of the disk.

VI. CONCLUSIONS AND FUTURE PROSPECTS

A. CONCLUSIONS

This paper summarizes the research progress of prediction methods on the crack propagation paths, mechanical models of open and breathing cracks, dynamic modeling methods and vibration behaviors of rotating blades. The main conclusions are summarized as follows:

(1) Accurately predicting the path of crack propagation can provide guidance for precise crack modeling. Prediction methods on the crack propagation paths mainly include TFEM, XFEM, meshless method, boundary method, and SBFEM. At present, for complicated blade structures, FEM, XFEM, and FEM-BEM (one of SBFEM) are mainly adopted.

(2) Modeling methods of open cracks mainly include the ERS method, which can adopt different-shape slots to simulated crack section, CCBB method, and local flexibility method from fracture mechanics. At present, local flexibility method is widely used where the local flexibility can be calculated by the fracture mechanics relations between the strain energy release rate and SIF, the Castigliano theorem, or fine-mesh FE techniques.
Breathing cracks can be simulated by bilinear spring, harmonic function, nonlinear contact model, developed crack element, and stress-based breathing crack model. In these models, bilinear spring model having high efficiency, nonlinear contact model having high accuracy, and stress-based breathing crack model having high accuracy and efficiency are widely adopted.

(3) Dynamic model of cracked blades can be established by LMM, SAM, and FEM using beam, shell, solid and hybrid elements. LMM and SAM are primarily used for qualitative fault mechanism or vibration characteristic analysis by simplifying the blade as one or multiple lumped mass points with massless springs (single DOF, or multiple DOFs), or by simplifying the blade as regular beam, plate, and shell structures, etc. FEM can accurately describe the complicated vibration behaviors of the actual cracked blade structures such as varying-thickness-twisted blade. In addition, FEM can also improve calculation efficiency using appropriate element such as beam, shell, solid and hybrid elements, or model dimension reduction.

(4) Frequency veerings and crossings caused by crack and centrifugal stiffening can be observed. Quantitative crack diagnosis for determining crack location and depth can be performed based on natural frequencies. The superharmonic resonance response is sensitive to small crack and the subharmonic resonance for deeper crack. The magnitude of the constant component and damping ratio caused by crack will increase with the increasing crack severity. The distortion of the phase plane portraits is more evident at super- and subharmonic resonances, and the phase portrait in axial direction may be more sensitive than that in bending direction. For the blisk structure, the blade crack can lead to strong localization of the forced response; some typical features such a double resonance peak can be observed. In order to better reveal the nonlinear vibration mechanism induced by cracks, it is necessary to study the change law of crack surface contact pressure, contact state, and time-varying stiffness induced by crack breathing.

B. FUTURE PROSPECTS

(1) Digital twins for crack evolution process should be focused on. ①Assessing fatigue damage and fatigue life estimation accurately under vibration-based loads is still a challenge. The effect of temperature field on crack growth and the vibration of cracked blades should also be considered. ② Based on the method of fracture mechanics and parameter identification, the variation laws of stiffness and damping parameters during crack evolution should be obtained. It is worth noting that crack growth can lead to an increase in damping, which can partially or completely suppress nonlinear effects. ③ The coupling effect of crack propagation and vibration should be considered to describe the crack-induced vibration during the whole life cycle process.

(2) Prognostic and health management of rotating blades should be paid attention to. ① Frequency and mode shape-based damage identification techniques have many limitations. A run time change in amplitude due to the propagating crack can help in structural health monitoring and preventive maintenance. However, a lot of efforts are required to develop a robust correlation between the crack behavior and the change in vibration amplitude, especially under the action of multiple excitation loads. ② Using the nonlinear effects for damage detection, it is necessary to eliminate other nonlinear effects (geometric nonlinearity, boundary nonlinearity, and other potential faults such as rubbing). ③ Adopting BTT to detect and predict blade crack, as well as employing artificial intelligence-based methods to diagnose crack depth and location.

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Declarations

Conflict of interest

Hui Ma is an associate editor for the *Journal of Dynamics, Monitoring and Diagnostics*, and he was not involved in the editorial review or the decision to publish this article. The authors declare that they have no conflict of interest.

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