Development of a Digital Model of a Gear Rotor System for Fault Diagnosis Using the Finite Element Method and Machine Learning

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Abstract

Geared-rotor systems are critical components in mechanical applications, and their performance can be severely affected by faults, such as profile errors, wear, pitting, spalling, flaking and cracks. Profile errors in gear teeth are inevitable in manufacturing and subsequently accumulate during operations. This work aims to predict the status of gear profile deviations based on gear dynamics response using the digital model of an experimental rig setup. The digital model comprises detailed CAD models and has been validated against the expected physical behaviour using commercial finite element analysis software. The different profile deviations are then modelled using gear charts, and the dynamic response is captured through simulations. The various features are then obtained by signal processing, and various ML models are then evaluated to predict the fault/no-fault condition for the gear. The best performance is achieved by an artificial neural network with a prediction accuracy of 97.5%, which concludes a strong influence on the dynamics of the gear rotor system due to profile deviations.

Keywords: Finite element modelling; Machine learning; Gear profile errors; Digital model; Geared-rotor system.

1. Introduction

Gear constitutes one of the most vital components used in mechanical systems. They are extensively used for transmitting motion and power from simple to very complex mechanical systems. With the continuously growing demands of gear systems, specifically in very high-speed applications, it is essential to achieve precise gearing action. The *fundamental law of gearing* is essential to ensure the smooth and continuous transmission of motion between gears. Despite the best manufacturing technologies and

best practices followed across the industry, it is impossible to obtain ideal involute tooth profiles, which are essential to abide as per law of gearing. Apart from that, the elastic deformation of gears during meshing and gear tooth wear add to further errors and deviations of gear tooth profiles. The gear profile deviations have an adverse effect on gear transmission errors (TE). It is defined as the difference between the actual position of the driven gear and the ideal position the gear would have occupied if the driving and driven gears were of perfect conjugate profile. It sets gear mesh in continuous contact without any deformation of gear mesh (Ghosh and Chakraborty, 2016).

Ghosh and Chakraborty (2016) mentioned noise, vibrations, and reduced efficiency in mechanical systems as source of transmission errors which mainly arise due to profile deviations, assembly errors, elastic deformations and wear and lubrication issues. TE arises due to various types of faults associated with gears. The gear faults can be broadly classified as operational faults, which are caused during gear operation and tooth profile faults, which may be present due to initial manufacturing errors or may occur due to operational faults, like wear. Mohammed and Rantatalo (2020) classified the gear operational faults into two major classes based on: (i) lubrication-related failures: overload failure and bending fatigue failure, which are independent of the system's lubrication and bound to occur with time; and wear, pitting and scuffing, which depend on the lubrication of the system that is lubrication directly impacts these failure by smoothening the interaction between gear tooth. and (ii) location-related failures: wear, pitting, scuffing and flank overload, which occur on the tooth flank, whereas root overload and bending fatigue failure occur at the tooth fillets near the root. The various types of operational defects arising in gears are: (i) Cracks and breakage occur by iniatiation of cracks at gear root, (ii) Gear tooth wear caused by wearing of gear tooth by subsequent operations, (iii) Pitting and spalling resulting in formation of large pits due to surface fatigue failure, (iv) Scuffing or scoring (adhesive wear), which is the surface failure occurring due to lack of lubrication. Figure 1.2 depicts various classes of tooth profiles and deviations in gears. The L_{α} correspond to the length over which profile deviations are evaluated. The profile deviations include: (i) *Total Profile Deviation* (F_{α}) : It refers to the distance between the actual profile trace, (ii) Profile Slope Deviation $(f_{H\alpha})$: It refers to the distance between the design profile line and the mean profile line, measured at the tip diameter, (iii) Profile Form Deviation ($f_{f\alpha}$): It refers to the distance

between the two facsimiles of the mean profile trace, to enclose the actual mean profile trace, (iv) *Profile Crown* (C_{α}) refers to modification given to the tooth surface with a small amount of curvature.



Figure 1 Gear tooth profile deviations: (a) Total profile deviation, (b) Profile slope deviation, (c) Profile form deviation and (d) Profile crown

Various researchers have tried exploiting tooth profile modifications and errors to attain a smoother gear dynamic. On one hand, gear tooth profile errors are inevitable, whereas tooth modifications are intentionally made to compensate for the degradation in performance due to profile errors. Tesfahunegn et al. (2009) studied tooth profile modifications at the tip to compensate for deflections under load, modelling them in linear and parabolic forms and establishing the strong effect of modification on gear strength. Ma et al. (2016) discussed the tooth profile modifications belonging to three classes: tip relief, root relief and whole tooth profile modification and presented the equation for modelling profile modification curve at the tip. Bonori and Pellicano (2007) analysed the non-linear vibration of spur gears in the

presence of manufacturing errors treating them stochastically and modelled composite profile error represented by first four harmonics of the Fourier Series as in equation 1

$$e_{PE} = \sum_{k=1}^{4} E_k \cos(k\widetilde{\omega}_m t - \gamma_k) \quad \text{with} \quad \widetilde{\omega}_m = \frac{\omega_m}{N_g N_p}$$
(1)

where E_k and γ_k are amplitudes and phases, N_g and N_p are the number of teeth on gear and pinion respectively, and ω_m is the gear mesh frequency. Mucchi et al. (2010) considered the profile deviations in the study of the dynamics of the gear pump and modelled them as a summation constant term depicting slope deviation and a sinusoidal term depicting form deviation. Fernandez et al. (2014) analysed gear dynamics incorporating tooth profile deviations and modelled as sinusoidal shapes with amplitude $f_{f\alpha}$ and f_r cycles as

$$e_{PE}(s) = f_{Ha} \frac{(s-s_0)}{(s_f - s_0)} + \frac{f_{fa}}{2} \sin\left\{2\pi f_r \frac{(s-s_0)}{(s_f - s_0)}\right\}$$
(2)

where, $f_{f\alpha}$ is the profile form deviation, $f_{H\alpha}$ is the profile slope deviation, *s* is the roll path length, measured along tangent to base circle, and s_0 and s_f correspond to the starting and end point on the involute. Zheng et al. (2023) established a model considering tooth profile deviations and modelling the tooth profile error as a series of cosine functions as

$$e_{PE} = \sum_{n=1}^{N} A_{ern} \cos\left(\frac{2\pi n\beta}{\theta_{g0}} + \varphi_{ern}\right)$$
(3)

where, A_{ern} and φ_{ern} represent the amplitude and phase of the *n*th cosine, respectively; and θ_{go} and β represent the angle at which the gear teeth turn from entering to exiting the meshing area and represent the position corresponding point on the tooth profile. The vibration-based condition monitoring and integration of machine learning in the same has been a trending topic of research. Mohammed and Rantatalo (2020) discussed the significance of condition-based monitoring (CBM) in reducing failures and unplanned stoppages in gear systems. Sharma and Parey (2016) discussed the various signal parameters that have been used for various fault predictions in gear systems. Researchers have tried deriving various analytical and finite element models to study the gear mesh dynamics, which is the main parameter to implement CBM in gear systems. Chen et al. (2020) analyzed time-varying mesh stiffness and transmission errors as key excitations in gear meshing through an analytical model validated against FEM results. Chen et al. (2021) developed an analytical model to study the effects of tooth profile modifications, wear, corner contact, and structural coupling on gear mesh stiffness.

Zheng et al. (2023) improved existing analytical models of gear mesh by directly incorporating tooth profile deviations, extended contact, and structural coupling along with profile deviation. Saxena et al. (2016) performed a modal analysis of a gear rotor system using ANSYS Workbench, obtaining mode shapes and corresponding frequencies. Tiwari (2018) discussed the detailed analysis of torsional and transverse vibration of rotor systems using one dimensional FEM elements. Visnadi et al. (2024) developed finite element model of a gear rotor system to study gear mesh stiffness variations due to cracks. Liu et al. (2022) discussed the integration of a digital twin (DT), which is a virtual representation of a physical object, process or system coupled via real-time data for CBM. Kritzinger et al. (2018) classified the concept of digital twin based on level of data integration between physical and virtual entity: (i) Digital model which does not use any form of automated data exchange, (ii) Digital shadow with one-way data flow and (iii) Digital twin with fully integrated flow of data.

Further, the integration of machine learning to automate CBM for fault diagnosis is being widely used. Praveenkumar et al. (2014) used a support vector machine model to detect faulty and non-faulty gears based on wear criterion by acquiring vibration signals. Gecgel et al. (2019) studied the performance of machine learning and deep learning models for fault classification in gear tooth profiles using analytically simulated data. Lupea and Lupea (2022) implemented multi-class classification and studied the performance of various classifier algorithms to analyse mounting faults in an experimental rotor system setup. Das et al. (2023) discussed the various works done, for fault identification in various elements of the rotor system using machine learning (ML). The literature review has been done to start with the transmission error and fault introduction in gears and then narrowed down to the profile deviation. Subsequently, the profile modelling and work performed on the CBM, digital twin and finally application of the ML.

Through this work, the aim is to develop the digital model of single-stage gearing, giving more flexibility to introduce different profile deviations and modelling them in a realistic manner and develop a machine learning model to predict the deviations based on the dynamic response obtained from the system. For the same, the existing experimental rig setup is used as a reference and step-by-step development, and validation of components is done. Figure 2 depicts the flowchart giving an insight into the step-by-step process followed for the study. The gear profiles are derived from a gear chart to exactly replicate them in the digital model, which is conventionally lacking in current work and has been achieved by fitting the harmonic series. The work also becomes crucial with respect to the industrial aspect, where

the coordinate measuring probe-based gear testers are used to benchmark the obtained gear profiles against the required standards. This becomes a time-consuming task in industries for quality testing, only considering three/four gear teeth for measuring deviations.



Figure 2 Flowchart concluding the procedure followed to develop the digital model

2. System Modelling

There are various assumptions which have been considered for subsequent development of the model and are mentioned as (i) the developed components have been considered as linear, isotropic and homogenous, (ii) loads applied are well within the elastic range of the components (iii) defects except profile deviation have not been considered in the CAD models, (iv) thermal considerations along with wear and tear during the operational aspect are ignored, and (v) a few components in the model have been replaced with their equivalent boundary conditions. Figure 3 depicts the experimental setup from which the numerical model is derived. Motor of 1 HP was used to power the system, which drives the input shaft connected via flexible coupling to the motor. The pinion is mounted on the input shaft, which further meshes with the output shaft. Deep groove ball bearings support both the input and output shaft at each of the ends. A magnetic brake is used in the model, which prevents free rotation of the gear and ensures continuous mesh between the pinion and the gear. The mountings are used to fix the model over the cast iron bed. The equivalent digital model mainly consists of the modelling of the following components: shafts, pinion and gears and replacing the rest of the components by equivalent boundary conditions. Tables 1,2 and 3 depict the attributes of each of the components. Figure 4 depicts the rendered CAD model images of the components.



Figure 3 Experimental rig setup

Sr. No.	Parameter	Particulars				
Material properties						
1	Material name	Steel				
2	Density	7850 kg/m ³				
3	Young's modulus, E	2.1×10 ¹¹ Pa				
4	Poisson's ratio, v	0.3				
Physical measurements						
5	Number of steps	2				
6	Number of segments	3				
7	Total length	295 mm				
8	Segment wise length	Segment 1: 70 mm				
		Segment 2: 190 mm				
		Segment 3: 35 mm				
9	Segment wise diameter	Segment 1: 10 mm				
		Segment 2: 12 mm				
		Segment 3: 10 mm				

Table 1 Specification of the shaft

Table 2 Pinion and gear parameters used in the experimental rig

Sr. No.	Parameter	Pinion	Gear
	Geometric properties		
1	Type of profile	Involute	Involute
2	Number of teeth	16	35
3	Module (mm)	2	2
4	Pressure angle (°)	20	20
5	Pitch circle diameter (mm)	32	70
6	Base circle diameter (mm)	30.07	65.778
7	Addendum (mm)	2	2
8	Dedendum (mm)	2.5	2.5
9	Total height of tooth (mm)	4.5	4.5
10	Addendum diameter (mm)	36	74
11	Dedendum diameter (mm)	27	65
12	Profile/addendum shift coefficient (x_{mc})	0	0
13	Root fillet radius (mm)	1.381	1.415
14	Clearance (mm)	0.5	0.5
15	Tooth thickness (mm)	3.1416	3.14229
16	Space width (mm)	3.1416	3.14229
17	Backlash (mm)	0	0
18	Bore diameter (mm)	12	12
	Material properties		
	Young's Modulus (E)	210 GPa	210 GPa
	Poisson's ratio (v)	0.3	0.3

Table 3 Highlights the bearing dimensions used in the experimental rig

Sr. No.	Parameter	Value (mm)
1	Inner race groove radius of curvature (<i>r_i</i>)	4.86
2	Outer race groove radius of curvature (r_o)	4.95
3	Ball diameter (D_b)	4.76
4	Bore diameter (<i>d</i>)	10
5	Outer diameter (D)	26
6	Bearing pitch diameter (<i>D_m</i>)	18



Figure 4 Depicts the CAD model of gear, pinion and shaft

3. Validation

This section discusses the validation of each component to ensure that it behaves analogously to its ideal physical equivalent. The components are validated based on their static and dynamic behaviour. The validations are done to ensure the geometry, material characteristics, mass and stiffness properties are in accordance with the ideal physical models. For the finite element simulations, various modules of ANSYS Workbench have been used.

3.1 Validation of the shaft

The static validation of the shaft is done by comparing the deflection of the shaft from FEM by making one end of the shaft cantilevered while applying the load on the other end against the analytical deflection as in Figure 5 to validate the geometrical and material properties. Table 4 summarizes the deviation for 3 loading conditions of the analytical and FEM deflections. The dynamic validation is performed to validate the natural frequency of the shaft, which is validated for the mass and stiffness properties.



All Dimensions in mm



Figure 5 Depicts the modelling of the shaft for the static validation

Load	Deflection from	Deflection from Castigliano's	%
applied (N)	ANSYS (mm)	theorem (mm)	Deviation
100	6.73	6.72	0.16%
200	13.46	13.43	0.22%
300	20.20	20.16	0.15%

Table 4 Depicts the deflection obtained from ANSYS and analytically

Modal analysis is used for the dynamic validation of the shaft. The analysis is performed for two end conditions (i) fix-fix ends of the shaft and (ii) Free-free ends of the shaft, ensuring the behaviour of the shaft remains validated against all other intermediate conditions. Figure 6(a) and 6(b) depict the analytical FE model and the simulation setup in ANSYS Workbench, respectively. The validation data is mentioned in Table 5, indicating the first 3 fundamental frequencies within 4% deviation.



Figure 6 (a) Depicts the FE model used for analytical modelling, (b) the 1st mode shape and frequencies obtained in both transverse planes

Sr. No.	Boundary condition	Mo no	ode Mode o. type	Natural frequency (Hz) from theoretical calculations	Natural frequency (Hz) from ANSYS simulation	% Deviation
	Fixed -	1	Transverse	510.05	513.06	0.59%
1	fixed -	2	Transverse	1484.74	1498.2	0.91%
IIXCu	lixeu –	3	Transverse	2869.04	2964.9	3.34%
		1	Transverse	664.58	669.67	0.77%
2	Free-free	2	Transverse	1669.73	1685.3	0.93%
		3	Transverse	3018.52	3138.5	3.97%

Table 5 Benchmarking A	ANSYS simulation	frequencies	against the a	inalytical	solution
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3.2 Validation of gear pair

The validation of the gear pair is done in 2 parts: first, the kinematic validation of the gear pair ensures proper meshing and correct modelling of the ideal involute gear tooth profile and then is followed by validating the meshing stiffness, which essentially takes care of the flexibility and contact. For simulation, the pinion is given a constant rotational velocity of 100 rad/s for a period of 0.063 s with a surface mesh size of 0.5 mm elements and the gear

ratio is calculated. The setup is depicted in Figure 7, where the gear and pinion are under mesh. The instantaneous gear ratio lies within 6% of the ideal gear ratio, which is mainly attributed to the contact ratio effect. The average gear ratio comes out to be 2.1870 against the ideal gear ratio of 2.1875, which accounts for 0.02% deviation. Figure 8 depicts the gear mesh mode, where gear teeth are taken as ideal involute profiles.



Figure 7 Setup for rigid dynamic analysis of gear pair in ANSYS Workbench



Figure 8 Variation of gear ratio during gear meshing obtained from the ANSYS simulation

For the mesh stiffness validation, analytical models are initially formulated, taking the stiffness of the gear tooth, pinion tooth and contact mesh stiffness into consideration, and then benchmarked with equivalent stiffness obtained from FEM. Figure 9 shows the schematic used for deriving the energy expressions and further development of the gear mesh stiffness formulations. The relevant dimensions of gears are obtained from Table 2. *X* and *Y* denote the Cartesian coordinate system; *X*' and *Y*' denote the transformed coordinate system where the Y' axis is aligned along the axis of the tooth, *dy*' denotes the elemental section thickness of the tooth at distance *y*' measured along the *Y*' axis, β is the involute angle or polar angle of the involute profile, λ is the angle between the start of the involute at the base circle and the line of symmetry passing through the tooth, α_c is the angle made by the force acting normally to the involute profile with the line perpendicular to the line of tooth symmetry, ψ denotes the angle made by the instantaneous normal to the involute with respect to the *Y* axis. *F* is the force acting at point C normal to the tooth, and $r_{inv}(\psi)$ is the radial distance from the origin to the instantaneous point on the involute curve. The various dimensions of each gear and pinion have been summarised in Table 6.

Parameter	Pinion	Gear
β_l	0°	O°
β_{u}	<i>4.29</i> °	<i>2.26</i> °
λ	<i>6.48</i> °	<i>3.43</i> °
R _b	15.04 mm	32.89 mm

Table 6 Various dimensional parameters for gear and pinion



Figure 9 Model used for determining tooth stiffness

The parametric equation for the involute profile is given as

$$x = R_b(\sin\psi + \psi\cos\psi), y = R_b(\cos\psi - \psi\sin\psi)$$
(4)

The width of the tooth is determined to be $e(\psi) = 2r_{inv}(\psi)\sin(\lambda - \beta)$. The strain energy and stiffness expressions are mentioned as

$$U_t = U_b + U_a + U_s \tag{5}$$

with

$$U_{b} = \int \frac{M^{2} dy'}{2EI};$$
 $U_{s} = \int \frac{S^{2} dy'}{2A_{r}G};$ $U_{a} = \int \frac{F^{2} dy'}{2AE};$

$$k_{st} = \frac{F^2}{2U_t} \tag{6}$$

where, U_b is the bending energy, U_s shear energy, U_a axial compression energy, E and I denote Young's modulus of the gear material, and P, M, and S denote the load, moment, and shear force, respectively. The integration is carried out using the Gauss quadrature due to the high level of complexity involved with the implicit expressions, and the convergence was observed with 120 Gaussian points. Figure 10 depicts the setup to determine the tooth

stiffness in ANSYS with a load applied at the tip of the gear. Table 7 compares the tooth stiffness for gear and pinion obtained analytically, and that from FEM using the force-deflection curve in Figure 11, indicating a deviation of 6.41%. This accounts for the portion of the tooth up to the involute profile, and the tooth fillet stiffness is disregarded in the analysis. The analysis also considers the gear body stiffness, which increases the overall stiffness.



Figure 10 Setup for tooth stiffness determination in ANSYS static structural workbench



Figure 11 Force-deflection curve for determining pinion and gear tooth stiffness

Table 7 Summary of pinion and gear tooth stiffness obtained analytically and ANSYS simulation

Particular	Pinion stiffness	Stiffness	
	analytically	from	%
	(N/mm)	ANSYS (N/mm)	Deviation
Pinion	457520.03	428210.52	6.41%
Gear	352612.75	328127.05	6.93%

Once the tooth stiffness is validated, the validation for contact mesh stiffness is performed. Marafona et al. (2021) stated that contact stiffness mainly refers to the stiffness related to the instantaneous point of contact of gears in mesh. Contact stiffness plays a vital role in determining the gear dynamics and, hence the overall characteristic of the gear rotor system and is greatly influenced by any minor deviations in tooth profile. Marafona et al. (2021) discussed three main approaches to calculate the gear tooth compliance, y_h , which are mentioned in equations (7) through (9).

 Hertzian and compression approach used as Hamilton Standard (Marafona et al., 2021):

$$y_h \approx \frac{4F}{\pi b} \left[\left(\frac{1 - v_1^2}{E_1} \right) + \left(\frac{1 - v_2^2}{E_2} \right) \right] \left[1 + \frac{\pi}{4} \right]$$

$$\tag{7}$$

2) Semi-empirical approach developed by Palmgren (Marafona et al., 2021):

$$y_{h} = \frac{4(1-v^{2})F}{\pi E_{12}b} \{1.10\frac{b^{2}E_{12}}{F}\}^{0.1} \qquad \text{with,} \qquad E_{12} = \frac{1}{2}\left(\frac{1}{E_{1}} + \frac{1}{E_{2}}\right)$$
(8)

3) Closed-form approach developed by Weber (Marafona et al., 2021):

$$y_{h} = \frac{2F}{\pi b} \left[\left(\frac{1 - v_{1}^{2}}{E_{1}} \right) \left\{ \ln \left(\frac{2h_{1}}{b_{H}} \right) - \left(\frac{v_{1}}{2(1 - v_{1})} \right) \right\} + \left(\frac{1 - v_{2}^{2}}{E_{1}} \right) \left\{ \ln \left(\frac{2h_{2}}{b_{H}} \right) - \left(\frac{v_{2}}{2(1 - v_{2})} \right) \right\} \right]$$
(9)
with, $b_{h} = \left[\frac{\left(\frac{4F}{\pi b} \right) - \left[\left(1 - v_{1}^{2} / E_{1} \right) + \left(1 - v_{2}^{2} \right) / E_{2} \right]}{\left[1 / \rho_{1} + 1 / \rho_{2} \right]} \right]^{\frac{1}{2}}$

where, b_h , is the half Hertz contact width, subscripts 1 and 2 denote the pinion and gear respectively, h_1 and h_2 are the distances on the pinion and the gear between the point of contact and the tooth centreline along the line of action and ρ_1 , ρ_2 are the curvature radii of the pinion and gear respectively, and E_1 , v_1 and E_2 , v_2 denote the Young's modulus and Poisson's ratio of pinion and gear material, respectively, and F is the force. The contact stiffness is then given by: $k_h = \frac{F}{y_h}$. The three models for mesh stiffness are evaluated using

equations 7 to 9. The contact stiffness using the Hertzian and compression approach is found to be 9.43×10^5 N/mm, which becomes constant and thus independent of force. The mesh stiffness variation obtained from Palmgren and Weber model is depicted in Figure 12(a) and 12(b) respectively. For FE modelling, the angle of action for the pinion is divided into 15 incremental steps, with the initial contact point to be taken as 0° and gradually an increment of 0.306° is given to the pinion and the contact stiffness is evaluated. The hub of the gear is fixed, and torque is applied on the pinion, which is free to rotate about its axis. Contact elements are deployed to capture the contact stiffness accurately. Figure 13 summarises the setup for the FE modelling that is used in the ANSYS environment. Figure 14 depicts the contact stiffness and contact area as the meshing occurs. The contact stiffness starts from an initial value of 1.86×10^7 N/mm at $\theta = 0^\circ$ and reaches a maximum value of 2.08×10^7 N/mm at an angular rotation value of $\theta = 2.76^\circ$, post which it starts decreasing and reaches a value of 1.91×10⁷ N/mm. Figure 15 compares the contact stiffness values from FEM against analytical models, where it agrees closest with the Weber model with a maximum deviation at $\theta = 2.76^{\circ}$ stands at a 9.48%. It is due to the FE model couples the stiffness of the gear body and leading to deviation from the ideal line contact at all instances during the meshing of the gears. Thus,

the overall stiffness of the meshing model is given by $k^{tp} = \frac{1}{1/k_1^{st} + 1/k_2^{st} + 1/k_h}$, where k_1^{st} is

the pinion tooth stiffness and k_2^{st} is the gear tooth stiffness. The maximum stiffness therefore obtained from FEM deviates by maximum 6.66% against the stiffness obtained analytically.



Figure 12 Contact stiffness-pinion angular rotation as obtained by the (a) Weber model (b) Palmgren model



Figure 13 Depicts the setup for gear mesh stiffness analysis in ANSYS workbench (a) Isometric view (b) Front view



Figure 14 Variation of contact stiffness with angular rotation of the pinion; the primary axis depicts the stiffness values, and the secondary axis depicts the contact area values



Figure 15 Comparison of analytical models and finite element models for contact stiffness

3.3 Sub-assembly and assembly validations

This section discusses the sub-assembly and system assembly validations. The validation performed ensures the correct modelling of contacts and boundary conditions, and accurate meshing of the gear pair. The modal analysis is carried out to validate the dynamic behaviour. The pinion and gear are taken to be rigid for sub-systems while flexible for final assembly validation, and the shaft is taken as a flexible body for both cases. For validating the finite element solid element data, the 1-D FE analysis is carried out. The 1-D FE analysis is carried out using 8 beam elements (Tiwari, 2018), neglecting the material damping of the shaft. The boundary conditions for the free-free case result in the shear force and the bending moment being zero at the free ends, while for fix-fix end conditions result in displacement and slope at the fixed end are set to zero for sub-system validation. Table 8 summarises the deviations of frequencies obtained from the solid model and the 1-D model using FEM for the subassemblies. For system validation, two frequencies are considered: the fundamental transverse and torsional frequencies. The fundamental transverse frequency is obtained experimentally (Deshpande & Tiwari, 2024), whereas the torsional frequency is obtained using 1-D FEM (Tiwari, 2018). The bearings and coupling ends are modelled as remote displacements with bending and rotation allowed. Figure 16 depicts the torsional mode of vibration for the system. Table 9 summarises the deviation of natural frequencies of the digital model against the equivalent physical/1-D FEM analytical model.

Particular	Boundary	Natural frequency (Hz)	icy (Hz) Natural	
	condition	from 1-D FEM	frequency (Hz)	Deviation
			ANSYS	
			simulations	
Shaft with	Fix-Fix	424.08	435.27	2.64%
Pinion	Free-Free	625.52	634.54	1.44%
Shaft with	Fix-Fix	273.7	279.4	2.08%
Gear		52(40	5 47 42	2.0.40/
	Free-Free	530.49	547.43	2.04%

Table 8 Represents the validation data for sub-systems

The deviations in the 1-D model and solid model FE simulations are attributed to the following reasons: (i) In the torsional case, with the 1-D model the gears are rigid bodies

whereas, in the 3-D model the gears are taken as flexible bodies to mesh, (ii) Cases with fixfix boundary condition have more deviation as in 3-D model several nodes are restricted from displacement, unlike the case in the 1-D model, thus adding to some artificial stiffness. Some possible deviations are expected of the physical system components with the 3-D model, which is mainly attributed to the following reasons: lack of homogeneity and isotropy in the 3-D models, tolerances involved in manufacturing and unbalances in the physical system.

Table 9 Benchmarking the fundamental natural frequency of the system obtained from 3-D model against the test rig and 1-D model.

Frequency Type	Fundamental natural frequency from test rig / 1-D FEM	Fundamental natural frequency (Hz) from	% Deviation
	(Hz)	ANSYS	
Transverse	234.00 (Experimentally)	217.45	7.07%
Torsional	6611.9 (Analytically)	6112.23	7.56%



Figure 16 A snip of modal analysis workbench depicting torsional frequency and mode shape

3.4 Bearing linearisation

Bearing forms an essential component of the experimental rig setup as the complex dynamics of the bearing has a major effect on the overall dynamics of the system. The inclusion of the bearing CAD model possesses two main challenges: the complex geometry of the bearing elements (such as balls, races and their eventual contact) making it computationally

expensive and the non-linear behaviour of the bearing further makes the analysis of the system complicated. Thus, a simplified approach has been used here to include the bearing dynamics in the operating force range of the model by linearising the bearing and treating it as a spring model. For the slow and moderate speeds, deep-groove ball bearings subjected to a radial load that causes the radial deflection only (axial deflection being zero), the force-deflection equation given by Palmgren (Harris and Kotzalas, 2006) is

$$\delta_r = 4.36 \times 10^{-4} \frac{Q_{\text{max}}^{2/3}}{D_b^{1/3} \cos a}$$
(13)

where, δ_r is the displacement (m) in the radial direction due to the maximum rolling element load Q_{max} (N), D_b is the diameter of the ball (m) and α is the contact angle (deg.). The value for the parameters is mentioned in Table 10. for the bearing are $\alpha = 0^{\circ}$ and $D_b = 4.764$ mm and Q_{max} expression is obtained using Stribeck's equation (Harris and Kotzalas, 2006) for the radial load, F_r . Considering a resisting torque of 0.5639 N-m, which is being implemented with the help of a magnetic brake on the gear shaft. The forces acting in the tangential and radial directions on bearing are obtained as in Table 10. The bearing is then linearised using equation 13 in the obtained force range and the stiffness values are mentioned in Table 11.

Table 10 Load acting on bearings

Bearing Location	Direction with respect to line of action	Load (N)
Input shaft	Tangential	7.65
Input shaft	Radial	2.93
Output shaft	Tangential	9.63
Output shaft	Radial	2.93

Sub-assembly	Parameter	Value (N/m)
Gear-Shaft	k_{gr} (Radial stiffness)	11934.94
	k_{gt} (Tangential stiffness)	17785.13
Pinion-Shaft	k_{pr} (Radial stiffness)	11934.94
	k_{pt} (Tangential stiffness)	16753.82

Table 11 Stiffness value obtained for bearings

4. Modelling of profile deviation and simulations

To model the profile deviations exactly, the 'web plot digitiser' app was used, which involves the coordinate extraction of actual gear profiles from the gear chart provided by the manufacturer. The required profile is grade 6 as per DIN standards. Any grade higher than this is considered faulty, and lower grades are considered non-faulty. The different profile deviations for each of the 40 fault cases are summarised in Table 12. For brevity, only the first 4 faults are depicted in Figure 18. The faults are assumed symmetrical on the tooth, and each of the gear teeth has the same profile to shorten the simulation time. Practically varying deviations can be seen as a superposition of various single deviations. The pinion and gear are bonded to the shaft. The coupling between the input shaft and the motor is replaced by a revolute joint at which a constant angular velocity of 14 Hz is applied, which acts as input power supplied to the system by the motor. A resisting moment of 0.5639 N-m is applied to replace the magnetic brake on the gear shaft. The bearings at the shoulders of the shaft are replaced by springs at 4 mm from the shoulders, as in Figure 17. Further, to ensure proper meshing of gears, a frictionless contact type is used between the gear and pinion teeth. The entire system is meshed with a mesh size of 1.5 mm with an additional contact mesh of 0.5 mm, ensuring proper meshing of gears by validating the gear ratio to be within 0.04% of the ideal gear ratio. The meshing between the gears is formulated using the augmented Lagrange solver method, ensuring efficient formulation of non-linear contact. The data is collected for 40 fault cases as per Table 13, using four deformation probes in Y and Z directions placed at 30 mm from the gear/pinion on respective shafts at a sampling rate of 1000Hz to accurately capture the meshing frequency. Figure 19 depicts the displacement data for the fault-1 case obtained from the simulation collected using four probes. Further, Table 14 benchmarks the simulation signal against the experimental signal (Deshpande and Tiwari, 2024) qualitatively, where cross-correlation is used to find the hidden patterns in the experimental signals corresponding to fault cases 11, 10, 9 and 8. An average correlation value of 0.52 is obtained; the deviations are likely due to: replacing the motor and magnetic brake by equivalent boundary conditions, simplified bearing dynamics, unintentional defects in the assembly of the experimental setup, noise in the data acquired through sensors and negligence of profile deviations in the pinion.



Figure 17 Model setup in ANSYS Workbench

Table 12 Depicts the	F_{α}	for	different	fault	cases
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Fault	F _α (μm)	Fault	F _α (μm)	Fault	F _α (μm)	Fault	Γ _α (μm)
Fault 1	16.5	Fault 11	8.6	Fault 21	4.3	Fault 31	11
Fault 2	10.7	Fault 12	5.8	Fault 22	4.5	Fault 32	`9.2
Fault 3	12.7	Fault 13	5.7	Fault 23	7.8	Fault 33	3.2
Fault 4	22.7	Fault 14	5.9	Fault 24	5.8	Fault 34	4.2
Fault 5	14	Fault 15	6.9	Fault 25	17.5	Fault 35	3.8
Fault 6	11.7	Fault 16	6.4	Fault 26	13	Fault 36	30
Fault 7	11.9	Fault 17	6.1	Fault 27	15.3	Fault 37	4
Fault 8	8.6	Fault 18	6.4	Fault 28	16	Fault 38	5
Fault 9	8.3	Fault 19	3.6	Fault 29	12.4	Fault 39	7.2
Fault 10	8.7	Fault 20	4.3	Fault 30	10	Fault 40	24.9



Figure 18 Profile deviations obtained from gear charts for 1st four fault cases

Table 13 Lists the names of the probes and data a	acquired
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Probe	Location
IP1	30 mm from the pinion towards the motor on the input shaft
IP2	30 mm from the pinion towards the free end side of the input shaft
OP1	30 mm from the gear towards the free-end side of the output shaft
OP 2	30 mm from the gear towards the magnetic brake on the output shaft



Figure 19 Depicts the displacement data collected using four probes for fault case 1

			experiment		
Fault Sensor		Correlation	Fault	Sensor	Correlation
		Value		00000	Value
	IP1Y	0.49	_	IP1Y	0.49
11	IP2Z	0.57	9	IP2Z	0.59
11	OP1Z	0.49	_	OP1Z	0.51
	OP2Y	0.52	_	OP2Y	0.51
	IP1Y	0.49	_	IP1Y	0.51
10	IP2Z	0.59	8	IP2Z	0.60
	OP1Z	0.51	_	OP1Z	0.50
	OP2Y	0.52	_	OP2Y	0.48

Table 14 The cross-correlation values of the signals obtained from simulations against the experiment

5. Machine Learning

Once the data is acquired white Gaussian noise is added to make the data more realistic and robust, and then various statistical parameters for condition monitoring are determined: Energy Ratio (ER), FM0, Crest Factor (CF), Skewness(S), FM4, NA4, RMS, Mean, M6A, M8A, Kurtosis(K) and Standard deviation (SD) (Sharma and Parey, 2016). The best signal to predict the fault state is then determined using the Mutual Information (MI). The MI is a measure from information theory that quantifies the amount of information one random

variable contains about another, and hence, it is crucial to determine the most informative features with respect to the target variable. The different signals are analysed, and a particular parameter with the maximum MI score across 8 signals is chosen to be a feature for the development of ML models. Table 15 lists the features along with their MI scores and the signal from which they are taken. For dimensionality reduction, further correlations and principal component analysis (PCA) are performed. Table 16 depicts a correlation matrix where M6A is eliminated based on its low MI score and high correlation with M8A. Figure 20 depicts the PCA analysis and the variance of 95% achieved with 8 components.

Feature	MI Score	Signal
ER	0.2449	OP1Z
FM0	0.2335	IP2Y
CF	0.1690	OP1Y
Skewness	0.1578	OP1Y
FM4	0.1382	IP2Z
NA4	0.1070	IP2Z
RMS	0.0896	OP1Z
Mean	0.0798	IP1Y
M8A	0.0698	IP1Z
Kurtosis	0.0643	IP1Y
SD	0.0643	IP2Z
M6A	0.0546	IP2Y
Total MI	1 /	1720
Content	1.4	t/27

Table 15 Final list of selected features

Table 16 Correlation matrix for the final set of selected features

Feature/Feature	Mean	SD	S	RMS	K	CF	FM4	M6A	M8A	FM0	ER	NA4
Mean	1.00	-0.03	-0.32	-0.31	0.26	0.12	0.08	0.24	0.03	-0.09	0.11	0.14
SD	-0.03	1.00	-0.14	0.51	0.03	0.21	-0.40	-0.05	-0.22	0.55	0.02	0.60
S	-0.32	-0.14	1.00	0.19	0.18	-0.01	0.22	0.18	0.07	-0.03	0.07	-0.03
RMS	-0.31	0.51	0.19	1.00	-0.02	-0.31	-0.20	-0.01	-0.08	0.36	-0.13	0.01
K	0.26	0.03	0.18	-0.02	1.00	0.37	0.06	0.96	-0.03	-0.31	-0.14	0.04
CF	0.12	0.21	-0.01	-0.31	0.37	1.00	-0.08	0.31	-0.05	-0.17	-0.25	0.45
FM4	0.08	-0.40	0.22	-0.20	0.06	-0.08	1.00	0.12	0.88	-0.17	0.21	-0.15
M6A	0.24	-0.05	0.18	-0.01	0.96	0.31	0.12	1.00	-0.02	-0.27	-0.15	-0.01
M8A	0.03	-0.22	0.07	-0.08	-0.03	-0.05	0.88	-0.02	1.00	-0.11	0.15	-0.04
FM0	-0.09	0.55	-0.03	0.36	-0.31	-0.17	-0.17	-0.27	-0.11	1.00	0.20	0.31
ER	0.11	0.02	0.07	-0.13	-0.14	-0.25	0.21	-0.15	0.15	0.20	1.00	0.16
NA4	0.14	0.60	-0.03	0.01	0.04	0.45	-0.15	-0.01	-0.04	0.31	0.16	1.00



Figure 21 Variational of the cumulative variance with components in PCA

Once the PCA components are obtained, the various ML models, namely, Logistic Regression, K-Nearest Neighbour (KNN), Kernel Support Vector Machine (Kernel SVM), Random Forest and the Artificial Neural Network (ANN) have been implemented and optimised for their best prediction accuracy over the test data, with the aim to carry out binary classification between fault/non-fault case. To ensure better generalisation of the models over the data, K-fold validation is used with K=4 folds. Figure 22 represents the performance of various ML models over the selected feature set, where ANN to be best performing with 97.5% prediction accuracy with the optimised parameters and logistic regression to be worst performing with 58.41% prediction accuracy. Table 17 further depicts the optimised hyperparameters identified for different models to get the maximum prediction accuracy, along with the optimised performance.

Model	% Accuracy	Optimized Parameters
Logistic Regression	58.41%	None
KNN	92.49%	K=6, where K is the tuning parameter with K nearest neighbours to be considered when classifying
Kernel SVM	90%	Kernel selection with the best performance for the RBF Kernel
Random Forest	90.23%	Number of decision trees: $n_{\text{estimators}} = 10$
ANN	97.50%	Optimiser: Adam, 2 hidden layers with 15 neurons each, Output layer activation function: Sigmoid, Batch size: 5, Epochs: 50

Table 17 Optimised parameters for various ML models



Figure 22 Illustrates the performance of various ML models over the test set

6. Conclusions

This work mainly involves studying the effect of profile deviations on gear tooth flanks and predicting them from the dynamic response of the system. The digital model of the system is developed by validating against the equivalent analytical component at each step. The system is dynamically validated against the experimental setup. The different profile deviations are then modelled in the gear in the CAD environment to exactly replicate the deviation present in the tooth of the experimental gears. The response of the system is then collected for different cases in the time domain, and different ML models are then used for binary classification of the system as a fault/no non-fault state using the twelve features extracted from the signal data. The hyperparameters for the ML models are tuned to achieve the maximum prediction accuracy, along with K-fold validation to ensure better generalisation and a check on overfitting with a limited set of data. The logistic regression model performs poorly, whereas models such as KNN, Kernel SVM with rbf-kernel and Random Forest are found to be performing with above 90% accuracy, indicating a strong non-linear relationship between the response and the defects, which is likely due to a change in contact non-linearity with the profile deviations. The best performance for the prediction accuracy is obtained using the ANN at 97.5%, which can capture the complex non-linear relationships by optimising the weights to minimise losses. This concludes that the response of a gear rotor

system is strongly dependent on the profile deviation. The future scope involves converting the digital model to a digital shadow, where real-time data integration can be done with the physical model to improve the digital model, ensuring the model is more realistic and robust for performance against the experimental rig setup, which may then be further extended to the concept of digital twin. However, the challenges include high computational resources, the requirement of large data samples and profile deviation modelling, the inclusion of fine dynamics and complex infrastructure setup for real-time data transfer.

Conflict of Interest statement

The authors declare no conflicts of interest.

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